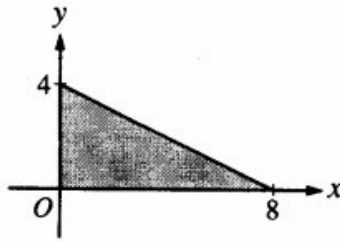AP Calculus AB
Review 05, Calculator Permitted

Complete all the following on notebook paper.

_____ 1.

The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x+2y=8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) 12.566   (B) 14.661   (C) 16.755   (D) 67.021   (E) 134.041

_____ 2.

Which of the following is an equation of the line tangent to the graph of $f(x)=x^4+2x^2$ at the point where $f'(x)=1$?

(A) $y=8x-5$
(B) $y=x+7$
(C) $y=x+0.763$
(D) $y=x-0.122$
(E) $y=x-2.146$

_____ 3.

Let $F(x)$ be an antiderivative of $\dfrac{(\ln x)^3}{x}$. If $F(1)=0$, then $F(9)=$

(A) 0.048   (B) 0.144   (C) 5.827   (D) 23.308   (E) 1,640.250

_____ 4.

If $0 \le k < \dfrac{\pi}{2}$ and the area under the curve $y=\cos x$ from $x=k$ to $x=\dfrac{\pi}{2}$ is 0.1, then $k=$

(A) 1.471   (B) 1.414   (C) 1.277   (D) 1.120   (E) 0.436

_____ 5.

If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.

_____ 6.

If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = \left(x^2 - 4\right)g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) f has relative minima at $x = -2$ and at $x = 2$.
(D) f has relative maxima at $x = -2$ and at $x = 2$.
(E)   It cannot be determined if f has any relative extrema.

_____ 7.

Let f be a function that is differentiable on the open interval $(1, 10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

   I. f has at least 2 zeros.
   II.   The graph of f has at least one horizontal tangent.
   III. For some c, $2 < c < 5$, $f(c) = 3$.

(A)   None
(B)   I only
(C)   I and II only
(D)   I and III only
(E)   I, II, and III

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

A table of values for a continuous function f is shown above. If four equal subintervals of $[0,2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8   (B) 12   (C) 16   (D) 24   (E) 32

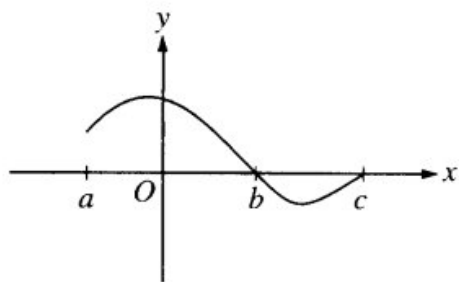Which of the following are antiderivatives of $f(x) = \sin x \cos x$?
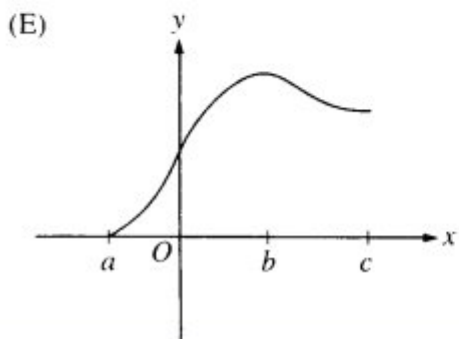
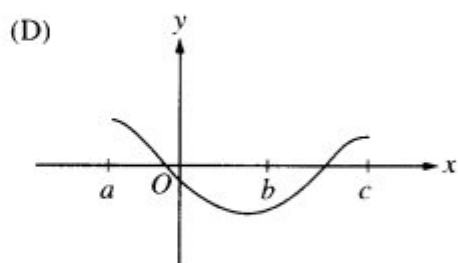I. $F(x) = \dfrac{\sin^2 x}{2}$
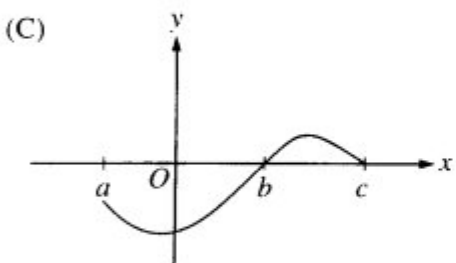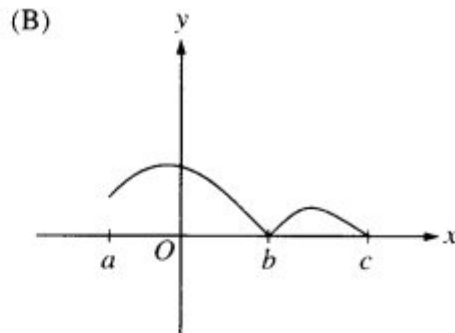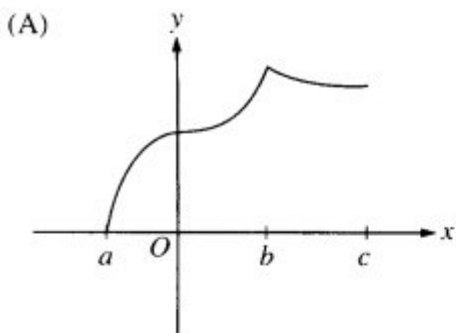
II. $F(x) = \dfrac{\cos^2 x}{2}$

III. $F(x) = \dfrac{-\cos(2x)}{4}$

(A) I only
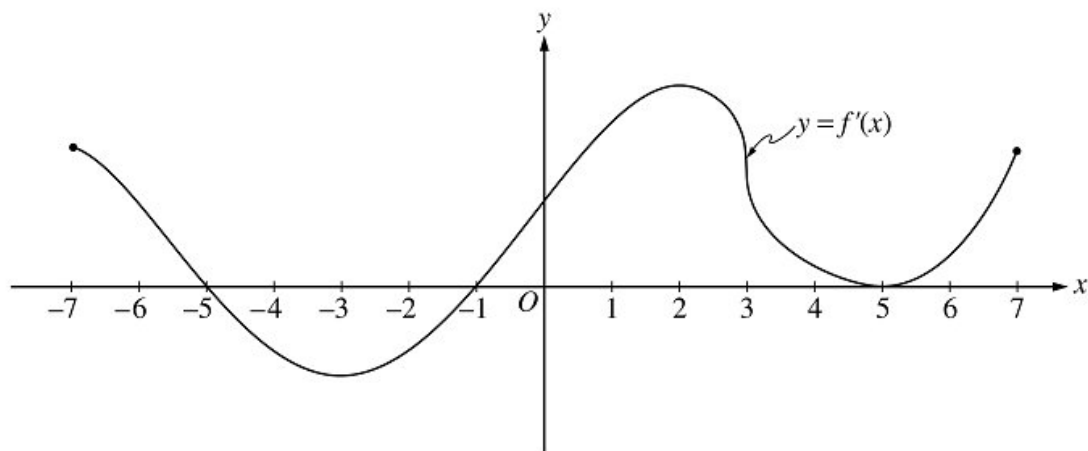
(B) II only

(C) III only

(D) I and III only

(E) II and III only

_____ 10.

Let $f(x) = \int_a^x h(t)\,dt$, where h has the graph shown above. Which of the following could be the graph of f?

(A)

(B)

(C)

(D)

(E)

11. 2000—AB3

The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

(a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

(b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

(c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

(d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

12. 2001-AB1
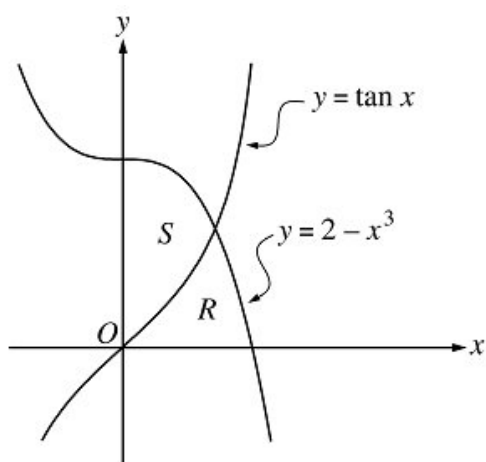
Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the x-axis.