1985 Calculus AB Solutions

1. D $\int_1^2 x^{-3}\,dx = -\frac{1}{2}x^{-2}\Big|_1^2 = -\frac{1}{2}\left(\frac{1}{4}-1\right) = \frac{3}{8}$.

2. E $f'(x) = 4(2x+1)^3\cdot 2$, $f''(1) = 4\cdot 3(2x+1)^2\cdot 2^2$, $f'''(1) = 4\cdot 3\cdot 2(2x+1)^1\cdot 2^3$,
$f^{(4)}(1) = 4!\cdot 2^4 = 384$

3. A $y = 3(4+x^2)^{-1}$ so $y' = -3(4+x^2)^{-2}(2x) = \dfrac{-6x}{(4+x^2)^2}$

Or using the quotient rule directly gives $y' = \dfrac{\left(4+x^2\right)(0)-3(2x)}{\left(4+x^2\right)^2} = \dfrac{-6x}{\left(4+x^2\right)^2}$

4. C $\int \cos(2x)\,dx = \frac{1}{2}\int \cos(2x)(2\,dx) = \frac{1}{2}\sin(2x)+C$

5. D $\lim\limits_{n\to\infty}\dfrac{4n^2}{n^2+10000n} = \lim\limits_{n\to\infty}\dfrac{4}{1+\dfrac{10000}{n}} = 4$

6. C $f'(x) = 1 \Rightarrow f'(5) = 1$

7. E $\int_1^4 \frac{1}{t}\,dt = \ln t\big|_1^4 = \ln 4 - \ln 1 = \ln 4$

8. B $y = \ln\left(\frac{x}{2}\right) = \ln x - \ln 2$, $y' = \frac{1}{x}$, $y'(4) = \frac{1}{4}$

9. D Since e^{-x^2} is even, $\int_{-1}^0 e^{-x^2}\,dx = \frac{1}{2}\int_{-1}^1 e^{-x^2}\,dx = \frac{1}{2}k$

10. D $y' = 10^{(x^2-1)}\cdot\ln(10)\cdot\frac{d}{dx}\left((x^2-1)\right) = 2x\cdot 10^{(x^2-1)}\cdot\ln(10)$

11. B $v(t) = 2t+4 \Rightarrow a(t) = 2 \therefore a(4) = 2$

12. C $f\left(g(x)\right) = \ln\left(g(x)^2\right) = \ln\left(x^2+4\right) \Rightarrow g(x) = \sqrt{x^2+4}$

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13. A $2x + x \cdot y' + y + 3y^2 \cdot y' = 0 \Rightarrow y' = -\dfrac{2x + y}{x + 3y^2}$

14. D Since $v(t) \geq 0$, distance $= \int_0^4 \left| v(t) \right| dt = \int_0^4 \left(3t^{\frac{1}{2}} + 5t^{\frac{3}{2}} \right) dt = \left(2t^{\frac{3}{2}} + 2t^{\frac{5}{2}} \right) \Big|_0^4 = 80$

15. C $x^2 - 4 > 0 \Rightarrow |x| > 2$

16. B $f'(x) = 3x^2 - 6x = 3x(x-2)$ changes sign from positive to negative only at $x = 0$.

17. C Use the technique of antiderivatives by parts:

BC topic

$u = x \qquad dv = e^{-x}\,dx$

$du = dx \qquad v = -e^{-x}$

$-xe^{-x} + \int e^{-x}\,dx = \left(-xe^{-x} - e^{-x} \right) \Big|_0^1 = 1 - 2e^{-1}$

18. C $y = \cos^2 x - \sin^2 x = \cos 2x$, $y' = -2\sin 2x$ Double angle; Lol!

19. B Quick solution: lines through the origin have this property.

Or, $f(x_1) + f(x_2) = 2x_1 + 2x_2 = 2(x_1 + x_2) = f(x_1 + x_2)$

20. A $\dfrac{dy}{dx} = \dfrac{1}{1 + \cos^2 x} \cdot \dfrac{d}{dx}(\cos x) = \dfrac{-\sin x}{1 + \cos^2 x}$

21. B $|x| > 1 \Rightarrow x^2 > 1 \Rightarrow f(x) < 0$ for all x in the domain. $\lim\limits_{|x| \to \infty} f(x) = 0$. $\lim\limits_{|x| \to 1} f(x) = -\infty$. The only option that is consistent with these statements is (B).

22. A $\int_1^2 \dfrac{x^2 - 1}{x + 1}\,dx = \int_1^2 \dfrac{(x+1)(x-1)}{x+1}\,dx = \int_1^2 (x-1)\,dx = \dfrac{1}{2}(x-1)^2 \Big|_1^2 = \dfrac{1}{2}$

23. B $\dfrac{d}{dx}\left(x^{-3} - x^{-1} + x^2 \right) \Big|_{x=-1} = \left(-3x^{-4} + x^{-2} + 2x \right) \Big|_{x=-1} = -3 + 1 - 2 = -4$

24. D $16 = \int_{-2}^{2} (x^7 + k)\,dx = \int_{-2}^{2} x^7\,dx + \int_{-2}^{2} k\,dx = 0 + \left(2 - (-2)\right)k = 4k \Rightarrow k = 4$

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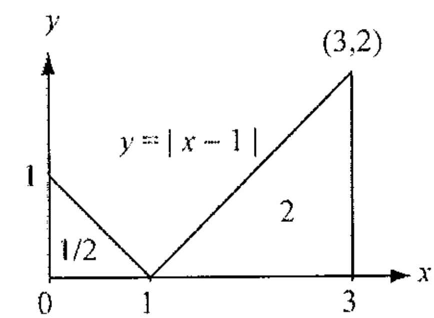
25. E $f'(e) = \lim_{h \to 0} \frac{f(e+h) - f(e)}{h} = \lim_{h \to 0} \frac{e^{e+h} - e^e}{h}$

26. E I: Replace y with $(-y)$: $(-y)^2 = x^2 + 9 \Rightarrow y^2 = x^2 + 9$, no change, so yes.
II: Replace x with $(-x)$: $y^2 = (-x)^2 + 9 \Rightarrow y^2 = x^2 + 9$, no change, so yes.
III: Since there is symmetry with respect to both axes there is origin symmetry.

27. D The graph is a V with vertex at $x = 1$. The integral gives the sum of the areas of the two triangles that the V forms with the horizontal axis for x from 0 to 3. These triangles have areas of 1/2 and 2 respectively.



28. C Let $x(t) = -5t^2$ be the position at time t. Average velocity $= \frac{x(3) - x(0)}{3 - 0} = \frac{-45 - 0}{3} = -15$

29. D The tangent function is not defined at $x = \pi/2$ so it cannot be continuous for all real numbers. Option E is the only one that includes item III. In fact, the functions in I and II are a power and an exponential function that are known to be continuous for all real numbers x.

30. B $\int \tan(2x)\,dx = -\frac{1}{2}\int \frac{-2\sin(2x)}{\cos(2x)}\,dx = -\frac{1}{2}\ln|\cos(2x)| + C$

31. C $V = \frac{1}{3}\pi r^2 h,\ \frac{dV}{dt} = \frac{1}{3}\pi\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right) = \frac{1}{3}\pi\left(2(6)(9)\left(\frac{1}{2}\right) + 6^2\left(\frac{1}{2}\right)\right) = 24\pi$

32. D $\int_0^{\pi/3} \sin(3x)\,dx = -\frac{1}{3}\cos(3x)\Big|_0^{\pi/3} = -\frac{1}{3}(\cos\pi - \cos 0) = \frac{2}{3}$

33. B f' changes sign from positive to negative at $x = -1$ and therefore f changes from increasing to decreasing at $x = -1$.

Or f' changes sign from positive to negative at $x = -1$ and from negative to positive at $x = 1$. Therefore f has a local maximum at $x = -1$ and a local minimum at $x = 1$.

34. A $\int_0^1 \left((x+8) - (x^3+8)\right)dx = \int_0^1 (x - x^3)\,dx = \left(\frac{1}{2}x^2 - \frac{1}{4}x^4\right)\Big|_0^1 = \frac{1}{4}$

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35. D The amplitude is 2 and the period is 2. no longer tested

$y = A\sin Bx$ where $|A| =$ amplitude $= 2$ and $B = \dfrac{2\pi}{\text{period}} = \dfrac{2\pi}{2} = \pi$

36. B II is true since $|-7| = 7$ will be the maximum value of $|f(x)|$. To see why I and III do not have to be true, consider the following: $f(x) = \begin{cases} 5 & \text{if} \quad x \le -5 \\ -x & \text{if} \quad -5 < x < 7 \\ -7 & \text{if} \quad x \ge 7 \end{cases}$

For $f(|x|)$, the maximum is 0 and the minimum is -7.

37. D $\lim\limits_{x\to 0} x\csc x = \lim\limits_{x\to 0} \dfrac{x}{\sin x} = 1$

38. C To see why I and II do not have to be true consider $f(x) = \sin x$ and $g(x) = 1 + e^x$. Then $f(x) \le g(x)$ but neither $f'(x) \le g'(x)$ nor $f''(x) < g''(x)$ is true for all real values of x.

III is true, since

$f(x) \le g(x) \Rightarrow g(x) - f(x) \ge 0 \ \Rightarrow \ \int_0^1 \big(g(x) - f(x)\big)\,dx \ge 0 \Rightarrow \int_0^1 f(x)\,dx \le \int_0^1 g(x)\,dx$

39. E $f'(x) = \dfrac{1}{x}\cdot\dfrac{1}{x} - \dfrac{1}{x^2}\ln x = \dfrac{1}{x^2}(1 - \ln x) < 0$ for $x > e$. Hence f is decreasing. for $x > e$.

40. D $\int_0^2 f(x)\,dx \le \int_0^2 4\,dx = 8$

41. E Consider the function whose graph is the horizontal line $y = 2$ with a hole at $x = a$. For this function $\lim\limits_{x\to a} f(x) = 2$ and none of the given statements are true.

42. C This is a direct application of the Fundamental Theorem of Calculus: $f'(x) = \sqrt{1 + x^2}$

43. B $y' = 3x^2 + 6x$, $y'' = 6x + 6 = 0$ for $x = -1$. $y'(-1) = -3$. Only option B has a slope of -3.

44. A $\dfrac{1}{2}\int_0^2 x^2\left(x^3 + 1\right)^{1/2} dx = \dfrac{1}{2}\cdot\dfrac{1}{3}\int_0^2 \left(x^3 + 1\right)^{1/2}\left(3x^2 dx\right) = \dfrac{1}{6}\left(x^3 + 1\right)^{3/2}\cdot\dfrac{2}{3}\Bigg|_0^2 = \dfrac{26}{9}$

45. A Washers: $\sum \pi\left(R^2 - r^2\right)\Delta y$ where $R = 2,\ r = x$

Volume $= \pi \int_0^4 \left(2^2 - x^2\right)dy = \pi \int_0^4 (4 - y)\,dy = \pi\left(4y - \frac{1}{2}y^2\right)\Big|_0^4 = 8\pi$