1985 AP Calculus AB: Section I

90 Minutes—No Calculator

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e).

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_1^2 x^{-3}\,dx =$

(A) $-\frac{7}{8}$ (B) $-\frac{3}{4}$ (C) $\frac{15}{64}$ (D) $\frac{3}{8}$ (E) $\frac{15}{16}$

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2. If $f(x) = (2x+1)^4$, then the 4th derivative of $f(x)$ at $x = 0$ is

(A) 0 (B) 24 (C) 48 (D) 240 (E) 384

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3. If $y = \frac{3}{4+x^2}$, then $\frac{dy}{dx} =$

(A) $\frac{-6x}{\left(4+x^2\right)^2}$ (B) $\frac{3x}{\left(4+x^2\right)^2}$ (C) $\frac{6x}{\left(4+x^2\right)^2}$ (D) $\frac{-3}{\left(4+x^2\right)^2}$ (E) $\frac{3}{2x}$

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4. If $\frac{dy}{dx} = \cos(2x)$, then $y =$

(A) $-\frac{1}{2}\cos(2x)+C$ (B) $-\frac{1}{2}\cos^2(2x)+C$ (C) $\frac{1}{2}\sin(2x)+C$

(D) $\frac{1}{2}\sin^2(2x)+C$ (E) $-\frac{1}{2}\sin(2x)+C$

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5. $\lim\limits_{n\to\infty} \frac{4n^2}{n^2+10{,}000n}$ is

(A) 0 (B) $\frac{1}{2{,}500}$ (C) 1 (D) 4 (E) nonexistent

6. If $f(x) = x$, then $f'(5) =$

(A) 0 (B) $\frac{1}{5}$ (C) 1 (D) 5 (E) $\frac{25}{2}$

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7. Which of the following is equal to $\ln 4$?

(A) $\ln 3 + \ln 1$ (B) $\frac{\ln 8}{\ln 2}$ (C) $\int_1^4 e^t \, dt$ (D) $\int_1^4 \ln x \, dx$ (E) $\int_1^4 \frac{1}{t} dt$

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8. The slope of the line tangent to the graph of $y = \ln\left(\frac{x}{2}\right)$ at $x = 4$ is

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1 (E) 4

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9. If $\int_{-1}^{1} e^{-x^2} dx = k$, then $\int_{-1}^{0} e^{-x^2} dx =$

(A) $-2k$ (B) $-k$ (C) $-\frac{k}{2}$ (D) $\frac{k}{2}$ (E) $2k$

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10. If $y = 10^{\left(x^2-1\right)}$, then $\frac{dy}{dx} =$

(A) $(\ln 10)10^{\left(x^2-1\right)}$ (B) $(2x)10^{\left(x^2-1\right)}$ (C) $\left(x^2-1\right)10^{\left(x^2-2\right)}$

(D) $2x(\ln 10)10^{\left(x^2-1\right)}$ (E) $x^2(\ln 10)10^{\left(x^2-1\right)}$

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11. The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

(A) 0 (B) 2 (C) 4 (D) 8 (E) 12

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12. If $f\left(g(x)\right) = \ln\left(x^2+4\right)$, $f(x) = \ln\left(x^2\right)$, and $g(x) > 0$ for all real x, then $g(x) =$

(A) $\frac{1}{\sqrt{x^2+4}}$ (B) $\frac{1}{x^2+4}$ (C) $\sqrt{x^2+4}$ (D) x^2+4 (E) $x+2$

13. If $x^2 + xy + y^3 = 0$, then, in terms of x and y, $\frac{dy}{dx} =$

(A) $-\frac{2x+y}{x+3y^2}$ (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

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14. The velocity of a particle moving on a line at time t is $v = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ meters per second. How many meters did the particle travel from $t = 0$ to $t = 4$?

(A) 32 (B) 40 (C) 64 (D) 80 (E) 184

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15. The domain of the function defined by $f(x) = \ln\left(x^2 - 4\right)$ is the set of all real numbers x such that

(A) $|x| < 2$ (B) $|x| \leq 2$ (C) $|x| > 2$ (D) $|x| \geq 2$ (E) x is a real number

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16. The function defined by $f(x) = x^3 - 3x^2$ for all real numbers x has a relative maximum at $x =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 4

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17. $\int_0^1 xe^{-x}\,dx =$

(A) $1-2e$ (B) -1 (C) $1-2e^{-1}$ (D) 1 (E) $2e-1$

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18. If $y = \cos^2 x - \sin^2 x$, then $y' =$

(A) -1 (B) 0 (C) $-2\sin(2x)$ (D) $-2(\cos x + \sin x)$ (E) $2(\cos x - \sin x)$

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19. If $f(x_1) + f(x_2) = f(x_1 + x_2)$ for all real numbers x_1 and x_2, which of the following could define f?

(A) $f(x) = x+1$ (B) $f(x) = 2x$ (C) $f(x) = \frac{1}{x}$ (D) $f(x) = e^x$ (E) $f(x) = x^2$

20. If $y = \arctan(\cos x)$, then $\frac{dy}{dx} =$

(A) $\frac{-\sin x}{1+\cos^2 x}$ (B) $-(\operatorname{arcsec}(\cos x))^2 \sin x$ (C) $(\operatorname{arcsec}(\cos x))^2$

(D) $\frac{1}{(\arccos x)^2 + 1}$ (E) $\frac{1}{1+\cos^2 x}$

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21. If the domain of the function f given by $f(x) = \frac{1}{1-x^2}$ is $\{x : |x| > 1\}$, what is the range of f?

(A) $\{x : -\infty < x < -1\}$ (B) $\{x : -\infty < x < 0\}$ (C) $\{x : -\infty < x < 1\}$

(D) $\{x : -1 < x < \infty\}$ (E) $\{x : 0 < x < \infty\}$

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22. $\int_1^2 \frac{x^2-1}{x+1}\,dx =$

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) $\frac{5}{2}$ (E) $\ln 3$

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23. $\frac{d}{dx}\left(\frac{1}{x^3} - \frac{1}{x} + x^2\right)$ at $x = -1$ is

(A) -6 (B) -4 (C) 0 (D) 2 (E) 6

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24. If $\int_{-2}^{2} (x^7 + k)\,dx = 16$, then $k =$

(A) -12 (B) -4 (C) 0 (D) 4 (E) 12

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25. If $f(x) = e^x$, which of the following is equal to $f'(e)$?

(A) $\lim_{h\to 0} \frac{e^{x+h}}{h}$ (B) $\lim_{h\to 0} \frac{e^{x+h} - e^e}{h}$ (C) $\lim_{h\to 0} \frac{e^{e+h} - e}{h}$

(D) $\lim_{h\to 0} \frac{e^{x+h} - 1}{h}$ (E) $\lim_{h\to 0} \frac{e^{e+h} - e^e}{h}$

26. The graph of $y^2 = x^2 + 9$ is symmetric to which of the following?

I. The x-axis
II. The y-axis
III. The origin

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

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27. $\int_0^3 |x-1|\,dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) 6

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28. If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0 \le t \le 3$ is

(A) -45 (B) -30 (C) -15 (D) -10 (E) -5

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29. Which of the following functions are continuous for all real numbers x?

I. $y = x^{\frac{2}{3}}$
II. $y = e^x$
III. $y = \tan x$

(A) None (B) I only (C) II only (D) I and II (E) I and III

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30. $\int \tan(2x)\,dx =$

(A) $-2\ln|\cos(2x)| + C$ (B) $-\frac{1}{2}\ln|\cos(2x)| + C$ (C) $\frac{1}{2}\ln|\cos(2x)| + C$

(D) $2\ln|\cos(2x)| + C$ (E) $\frac{1}{2}\sec(2x)\tan(2x) + C$

31. The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

(A) $\frac{1}{2}\pi$ (B) 10π (C) 24π (D) 54π (E) 108π

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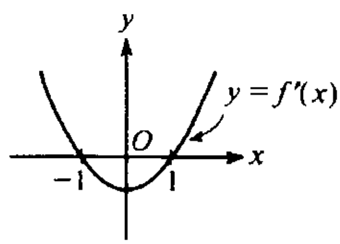
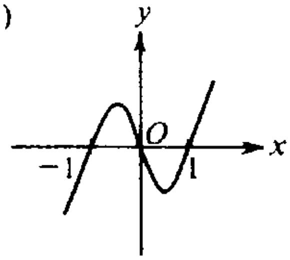
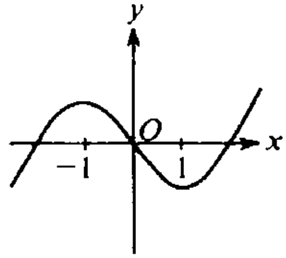
32. $\int_0^{\frac{\pi}{3}} \sin(3x)\,dx =$

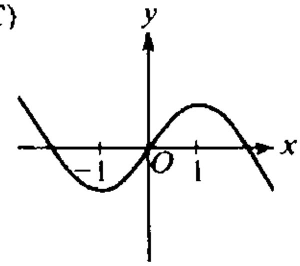
(A) -2 (B) $-\frac{2}{3}$ (C) 0 (D) $\frac{2}{3}$ (E) 2

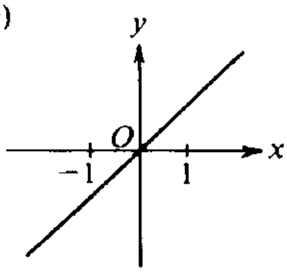


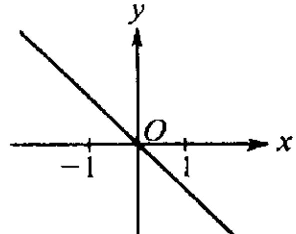
33. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

(A)


(B)


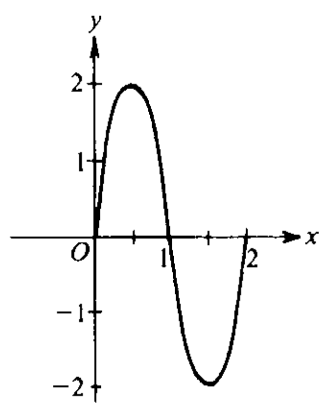
(C)


(D)


(E)


34. The area of the region in the <u>first quadrant</u> that is enclosed by the graphs of $y = x^3 + 8$ and $y = x + 8$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) $\frac{65}{4}$



35. The figure above shows the graph of a sine function for one complete period. Which of the following is an equation for the graph?

(A) $y = 2\sin\left(\frac{\pi}{2}x\right)$ (B) $y = \sin(\pi x)$ (C) $y = 2\sin(2x)$

(D) $y = 2\sin(\pi x)$ (E) $y = \sin(2x)$

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36. If f is a continuous function defined for all real numbers x and if the maximum value of $f(x)$ is 5 and the minimum value of $f(x)$ is -7, then which of the following must be true?

I. The maximum value of $f(|x|)$ is 5.
II. The maximum value of $|f(x)|$ is 7.
III. The minimum value of $f(|x|)$ is 0.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

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37. $\lim_{x \to 0} (x \csc x)$ is

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

38. Let f and g have continuous first and second derivatives everywhere. If $f(x) \leq g(x)$ for all real x, which of the following must be true?

I. $f'(x) \leq g'(x)$ for all real x
II. $f''(x) \leq g''(x)$ for all real x
III. $\int_0^1 f(x)\,dx \leq \int_0^1 g(x)\,dx$

(A) None (B) I only (C) III only (D) I and II only (E) I, II, and III

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39. If $f(x) = \dfrac{\ln x}{x}$, for all $x > 0$, which of the following is true?

(A) f is increasing for all x greater than 0.
(B) f is increasing for all x greater than 1.
(C) f is decreasing for all x between 0 and 1.
(D) f is decreasing for all x between 1 and e.
(E) f is decreasing for all x greater than e.

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40. Let f be a continuous function on the closed interval $[0,2]$. If $2 \leq f(x) \leq 4$, then the greatest possible value of $\int_0^2 f(x)\,dx$ is

(A) 0 (B) 2 (C) 4 (D) 8 (E) 16

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41. If $\lim_{x \to a} f(x) = L$, where L is a real number, which of the following must be true?

(A) $f'(a)$ exists.
(B) $f(x)$ is continuous at $x = a$.
(C) $f(x)$ is defined at $x = a$.
(D) $f(a) = L$
(E) None of the above

42. $\dfrac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt =$

(A) $\dfrac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}-5$ (C) $\sqrt{1+x^2}$

(D) $\dfrac{x}{\sqrt{1+x^2}}-\dfrac{1}{\sqrt{5}}$ (E) $\dfrac{1}{2\sqrt{1+x^2}}-\dfrac{1}{2\sqrt{5}}$

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43. An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$ (B) $y = -3x + 1$ (C) $y = 2x + 10$

(D) $y = 3x - 1$ (E) $y = 4x + 1$

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44. The average value of $f(x) = x^2\sqrt{x^3+1}$ on the closed interval $[0, 2]$ is

(A) $\dfrac{26}{9}$ (B) $\dfrac{13}{3}$ (C) $\dfrac{26}{3}$ (D) 13 (E) 26

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45. The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the y-axis. The volume of the solid generated is

(A) 8π (B) $\dfrac{32}{5}\pi$ (C) $\dfrac{16}{3}\pi$ (D) 4π (E) $\dfrac{8}{3}\pi$