# 2002 AP Statistics Exam Multiple Choice Solutions

**1. Answer: (D)**

(A) is FALSE → There are not necessarily more subjects available for either experiments or observational studies. There are probably more subjects available for observational studies if anything because people are more apt to be alright with someone observing them than to take place in an experiment.
(B) is FALSE → In anything, ethical constraints might prevent certain large scale experiments from taking place, not observational studies.
(C) is FALSE → On average, experiments tend to be more costly than observational studies.
(D) is TRUE → Only through a well-designed experiment can we determine a causal relationship between our variables whereby in an observational study, we can only determine if there is an association between our variables.
(E) is FALSE → You can do significance tests on any data, whether it comes from an observational study or an experiment.

**2. Answer: (B)**

Choices (A) and (E) are out from the get-go because the null hypothesis must have $p = 0.05$ in it. Since we are checking to see if the manufacturer is wrong (and the manufacturer says that it is "no more than 0.05"), we must be testing for the idea that the proportion is more than 5% or $H_a$: $p > 0.05$.

Note: Our parameter of interest in this example is $p$, the true proportion of balloons that burst when inflated to a diameter above 12 inches. This is because this is categorical data…the balloons either burst or don't.

**3. Answer: (C)**

If you want to see how these Lauren performed in a relative sense and the scores follow a normal distribution, then you may use z scores:

1st exam: $z = \dfrac{x-\mu}{\sigma} = \dfrac{85-75}{10} = 1$ | 2nd exam: $z = \dfrac{x-\mu}{\sigma} = \dfrac{85-70}{15} = 1$

Since the $z$ scores are the same [both times she scored exactly one standard deviation above the rest of the class], she scored about equally as well on both exams. The class size and correlation between the two scores is irrelevant for this question.

**4. Answer: (A)**

Recall that when you are setting up a simulation you must **disregard the values of the numbers** and simply treat them as non-numerical objects. Since 30% of all subscribers watch the shopping channel at least once a week, make 30% of the 10 numbers represent those people and the other 70% of the 10 numbers represent the other people. The numbers do not represent quantities, they only represent outcomes.

**5. Answer: (D)**

There are a couple different ways you can do this:

Method 1: Figure out the expected number of sweatshirts that are sold:

$$E(x) = \mu = \sum x_i p(x_i) = (0 * 0.3) + (1 * 0.2) + (2 * 0.3) + (3 * 0.1) + (4 * 0.08) + (5 * 0.02) = 1.52$$

Now since each sweatshirt is sold for $25, they should expect to make:

$$\$25 \times 1.52 = \$38$$

Method 2: Convert the original data from number of sweatshirts to amount of money earned:

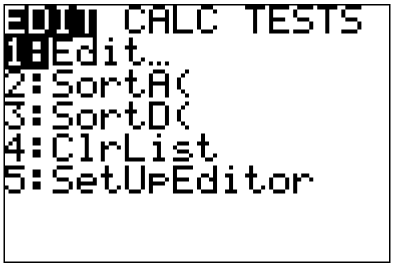
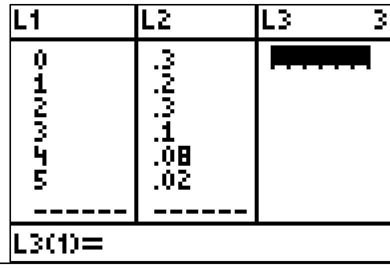
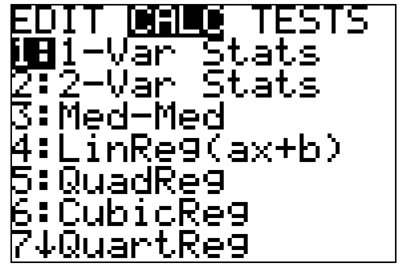
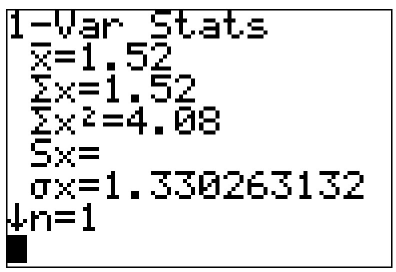
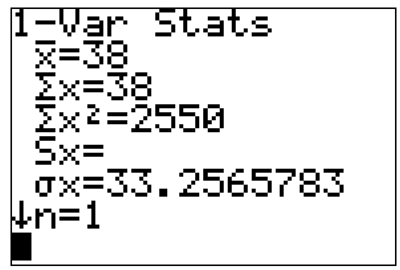
| Money per sweatshirt *x* | $0 | $25 | $50 | $75 | $100 | $125 |
|---|---|---|---|---|---|---|
| P(*x*) | 0.3 | 0.2 | 0.3 | 0.1 | 0.08 | 0.02 |

Now do the same thing to find the expected revenue:

$$E(x) = (0 * 0.3) + (25 * 0.2) + (50 * 0.3) + (75 * 0.1) + (100 * 0.08) + (125 * 0.02) = \$38.00$$

Keep in mind, though, that you don't have to do these by hand unless it's a free response question, and even then you need only show the formulas for the purpose of presentation.

If it's a multiple choice question like this, just do it in the calculator:

<table>
<tr>
<td>1. Press STAT and press ENTER.<br></td>
<td>2. Enter all the x values into $L_1$ and all the probabilities into $L_2$<br></td>
<td>3. Press STAT → CALC → 1-Var-Stats<br></td>
</tr>
<tr>
<td>3. Now tell it to do 1-Var Stats on $L_1$ with a frequency of $L_2$.<br></td>
<td>4. Press ENTER and the $\overline{x}$ is your mean (expected value) and $\sigma_x$ is your standard deviation.<br></td>
<td>4. You can also do this from the perspective of money… same exact process.<br></td>
</tr>
</table>

**6. Answer: (E)**

Recall that the correlation coefficient ($r$) is <u>not</u> affected by multiplying, adding, subtracting, or dividing (any linear transformation does <u>not</u> affect the correlation coefficient). The only thing that could affect the correlation coefficient was if you were to do some sort of non-linear transformation such as taking the log, taking the square root, or squaring every data value. So it remains the same.

**7. Answer: (A)**

Since a constant is being added to every data value, the mean will be affected but the standard deviation will not…

$$\mu = 47 + 4 = 51.$$

$$\sigma = 14$$

Now, had the question said that we, say, multiply every number by 2, then both the mean and the standard deviation would be affected:

$$\mu = 47 \times 2 = 94$$

$$\sigma = 14 \times 2 = 28$$

Also, had the question given you the variance and doubled every data value, that would have changed the answer as well. Pretend that the variance is 14 and we multiply every value in the data set by 2. You have to square the constant because the variance is in square units:

$$\text{Variance} = \sigma^2 = 2^2 \times 14 = 56$$

**8. Answer: (A)**

Immediately (E) is not the correct answer because even though the sample size is small ($n$ = 11), the question states that the data appears 'unimodal and symmetric' in which case the small sample size is no big deal. Recall that for a one-sample t-interval the degrees of freedom are $n$ – 1. On the t-table, look up the critical value for 95% confidence and $n$ – 1 = 11 – 1 = 10 degrees of freedom and you get 2.228:

| df | | | | | |
|---|---|---|---|---|---|
| 10 | 1.372 | 1.812 | 2.228 | 2.359 | 2.764 |
| : | : | : | : | : | : |
| | 80% | 90% | 95% | 96% | 98% |

Confidence Level $C$

The formula one sample t-interval is:

$$\bar{x} \pm t^*_{n-1} \times \frac{s}{\sqrt{n}} \rightarrow 25.5 \pm 2.228 \times \frac{3.01}{\sqrt{11}}$$

**9. Answer: (E)**

This guy is going to increase the sample size in hopes of fixing one of the following problems:

(A) is FALSE → Non-response bias is when you ask the question to people and they blatantly refuse to respond. The more people you ask, the more who will refuse to respond.
(B) is FALSE → Confounding variables are variables that you did not consider and that affect the response variable, making it difficult to ascertain the true effect. Increasing the sample size does not get rid of these. Only a well-designed experiment can help minimize the effects of confounding variables.
(C) is FALSE → The interviewer effect will not diminish as more people are surveyed. The interviewer effect is when the person conducting the interview (their demeanor, appearance, etc) makes it to where it is difficult to respond honestly. It is a form of response bias (i.e. anything that biases your response)
(D) is FALSE → You can not alter the variability in the population. There is no variability in the population. Variability only refers to sampling.
(E) is TRUE → As you increase the sample size, sampling variability (the degree to which statistics will differ from sample to sample) will decrease always. So, the standard deviation of the sampling distribution of the sample proportion will also decrease. Its formula is as follows:

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{pq}{n}}$$

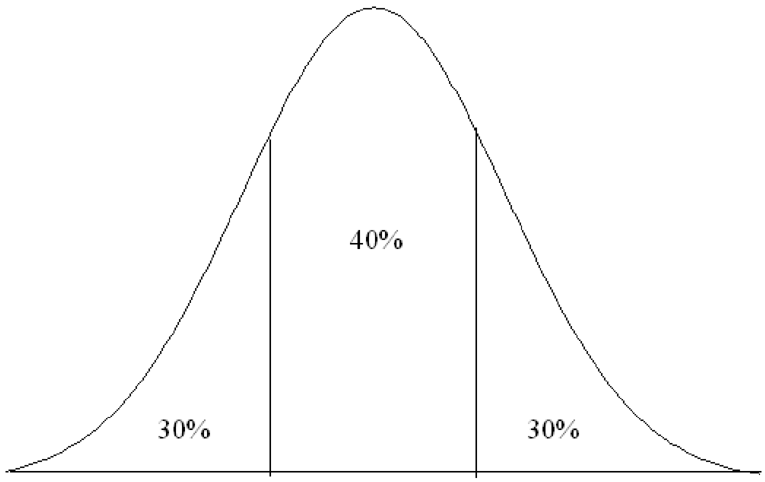
Notice that if we were to increase the sample size, this quantity would invariably decrease.

**10. Answer: (D)**

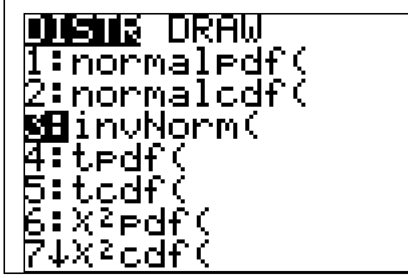
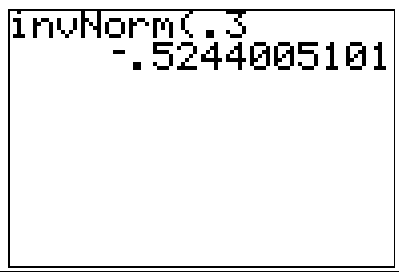
Since this question asks for the ‘shortest distance’ that will still give you 4,000 shellfish, it must be centered about the mean. Also, since there are 10,000 shellfish in the population, the question is asking for the shortest distance that will contain:

$$\frac{4,000}{10,000} = 40\% \text{ of the shellfish}$$

Using the ghetto-looking picture below, you realize that you have the area under the curve and you need the values on the *x*-axis:




In this case, use the TI83 to find both values (although you will notice that you need only one):

| 1. Press 2nd → VARS → invNorm( | 2. The command is invNorm(area to left) = $z$ |
|---|---|
|  |  |

Now, just solve for the missing value $x$:

$$z = \frac{x - \mu}{\sigma}$$

$$-0.5244 = \frac{x - 10}{0.2}$$

$x$ = 9.895 cm.

**11. Answer: (B)**

This is asking for a component of the $\chi^2$ test for independence. Look at wording in the question: "If the null hypothesis of no association between level of education and employment status is true…". This is asking for the expected value under the assumption of independence. The formula for this value is:

$$\text{Expected} = \frac{\text{Row} \times \text{Column}}{\text{Grand}} = \frac{92 \cdot 82}{157}$$

**12. Answer: (B)**

1. Since the data is quantitative (we are measuring the <u>mean</u> number of units assembled per employee), the goodness of fit test is irrelevant because that deals with categorical data.

2. Since there are 2 groups, we have to look for pairing. There is no evidence to believe that the experimental units are being paired in any way (no matching in the question).

3. The sampling procedure seems to indicate that the experimental units were sampled independent from one another. So he should use a two-sample *t*-test.

<u>Note</u>: Had there only been one group of employees and you had to distinguish between whether to use a *z*-test or a *t*-test, you have to see if the population standard deviation is known or not. If it is, use a *z*-test. If not, use a *t*-test.

**13. Answer: (A)**

This question harps at the different qualities of confidence intervals that you have to know for this exam:

(A) is TRUE → As your confidence level goes up, so too does the width of the confidence interval [a 99% CI is wider than a 95% CI which is wider than a90% CI, etc]
(B) is FALSE → It contradicts what I just said for choice (A).
(C) is FALSE → Sample size is not the only thing that affects the width of a confidence interval. Also, the question makes no mention of the sample size. However, if the question had not changed the confidence level, but did change the sample size, a <u>larger</u> sample size will yield a <u>narrower</u> confidence interval and thus more precision.
(D) is FALSE → If your sample is biased, there is no way you should be performing any type of statistical inference. Period.
**(E)** is FALSE → Whether you use a *z*-interval or a *t*-interval depends on whether you know the population standard deviation.

**14. Answer: (D)**

Let's go through these one at a time:

(A) is TRUE → Range is measured as max – min and in this case, the maximum and minimum values for both boxplots look about the same.

(B) is TRUE → The IQR is measured as Q3 – Q1:

For Data set I: Q3 – Q1 = 45 – 30 = 15
For Data set II: Q3 – Q1 = 50 – 35 = 15

(C) is TRUE → The median is measured by the center line in the boxplot, which for data set I is at approximately 35 and for data set II is approximately 45. Keep in mind that on a boxplot, this is a measure of the median, not the mean. The only time that this could be considered a measure of the mean would be if the data set looked approximately symmetrical at which time the mean and median would be approximately equal. Also, you can tell from this picture what the shape of each data set would be. Data set I looks slightly skewed right (in which case the mean would be above the median) and data set II looks slightly skewed right (in which case the mean would be less than the median).

(D) is FALSE → We have no idea what the sample sizes are for each one of these data sets. You cannot tell the sample sizes solely from box-and-whisker plots.

(E) is TRUE → The first quartile for data set II is approximately 35. At the first quartile, 75% of all data points are above that value (and 25% are below it). For data set I, the median is at about 35 as well. The median is the place where 50% of the data points are below it and 50% of the data points are above it. So, the statement "about 75% of the values in data set II are greater than about 50% of the data values in data set I" must be true.

**15. Answer: (D)**

What is the difference between each of the sampling procedures?

(A) **Cluster** sampling is when you take a random group that is already relatively heterogeneous (mixed) on the variable of interest. For example, we just grab my 1st period class as a representative sample of all my stats classes because we felt that they were a good mix on the variable of interest.

(B) **Convenience** sampling is a very bad thing. It is when you solely grab those people that are conveniently located to you. Imagine if I took a survey of all teachers during our weekly meeting to see how all teachers felt about the administration. This would not work too well because I don't know if their opinions are representative of all opinions. It's just convenient for me because they're right there.

(C) **Simple random** sampling is when we simply randomly sample from our population of interest. If we used a random number generator, or put names in a hat, we would be conducting a simple random sample

(D) **Stratified random** sampling is when we break people off into similar groups and then within each group (or strata), we sample people randomly. We want to divide the subjects into strata such that the people are as alike as possible on the variable of interest. This question is an example of stratified random sampling because they broke the class off into freshmen, sophomores, juniors, and seniors and then selected students from each stratum.

(E) **Systematic** sampling is when you take every $n^{th}$ person. If I were to stand in the hall area of a school and sample every, say, $5^{th}$ person, this would be systematic sampling.

**16. Answer: (E)**

Jason wants to determine how age and gender are <u>related</u> to political party preference:

(A) is FALSE → This is for bunch of reasons we are about to discuss.
(B) is FALSE → Sample <u>size</u> is not important. It's about quality, not quantity. A good small sample is better than a big crappy sample.
(C) is FALSE → Equal sample sizes are <u>not</u> required for your sample to be representative.
(D) is FALSE → The question says that he <u>did</u> choose the samples randomly. Liars.
(E) is TRUE → If he's trying to see the effect of political party preference on gender and age group, he has done a stupid thing. He has selected two independent samples that differ on <u>both</u> variables. Now, since they are different genders and different ages, he cannot tell if the difference in political preference is due to the difference in age groups or the difference in genders. In other words, they are confounded

**17. Answer: (B)**

17. A residual is always measured as A – P = Actual – Predicted. Since you are not given a predicted value, you have to plug the *x* value into the equation in order to get the predicted value:

$$\hat{y} = 16.6 + 0.65 \times 20 = 29.6$$

Actual – Predicted = 25 – 29.6 = –4.60.

Also, let's look at the slope and intercept (just for fun):

<u>Slope</u>: We predict, on average, that for each additional month of age, the child gains an additional 0.65 pounds of weight.

<u>Intercept</u>: We predict, on average, that when a baby is zero months old, he will weigh 16.6 pounds. This is actually describing the weight of the baby at birth, so it makes sense contextually. It is possible, though, that the data was not gathered from values that stretched down to zero so it is probably <u>extrapolation</u> to interpret this value.

**18. Answer: (E)**

Recall that the *t*-distribution is used when the standard deviation for the population is unknown. Here are the properties of the *t*-distribution that you may have to know:

1. It is symmetric and centered at zero
2. It is indexed by degrees of freedom
3. It is shorter and has more area in the tails than the z-distribution
4. A *z*-distribution is a *t*-distribution with infinite degrees of freedom
5. As degrees of freedom increase, the *t*-distribution approaches a *z*-distribution and thus has less variability as these degrees of freedom go up.

So…

I. The *t*-distribution is symmetric → TRUE [see above]

II. The *t*-distribution with *k* degrees of freedom has smaller variance than the *t*-distribution with *k* + 1 degrees of freedom → FALSE [A *t*-distribution with *k* degrees of freedom has <u>more</u> variance than the *t*-distribution with *k* + 1 degrees of freedom]

III. The *t*-distribution has a larger variance than the standard normal (*z*) distribution → TRUE [A *t*-distribution by definition has more variability than a *z* distribution.]

**19. Answer: (B)**

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19. The values given in the question are observed values for the 60 people who were sampled:

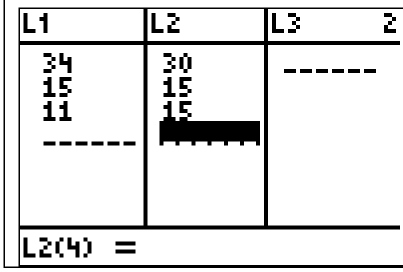
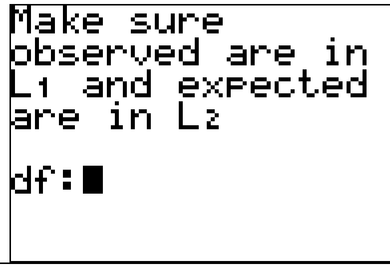
| Brown Eyes | Green Eyes | Blue Eyes |
|---|---|---|
| 34 | 15 | 11 |

It is hypothesized, though, that half of all people have brown eyes and the other half are evenly divided between green eyes and blue eyes. By that accord, the expected counts for these *n* = 60 people are as follows:

| Brown Eyes | Green Eyes | Blue Eyes |
|---|---|---|
| $60\times0.5=30$ | $60\times0.25=15$ | $60\times0.25=15$ |

$$\chi^2=\sum\frac{(\text{Observed}-\text{Expected})^2}{\text{Expected}}=\frac{(34-30)^2}{30}+\frac{(15-15)^2}{15}+\frac{(11-15)^2}{15}=1.6$$

Again, this is one of those questions you can do in the calculator. Keep in mind, though, that the goodness of fit test will <u>not</u> work if you try to put the data in a matrix and perform it like you would the tests for independence and homogeneity. You have to do this using the program I gave you:

| 1. Make sure all observed values are in $L_1$ and all expected values in $L_2$ | 2. Press PRGM and run the CHIGOF program | 3. Let it know that there are 2 degrees of freedom (#categories – 1) and you're done! |
|---|---|---|
|  |  |  |







**20. Answer: (B)**

Since we are attempting to <u>compare</u> two distributions, you have to be careful what you decide is/isn't an "appropriate" display:

(A) **Back-to-back stem and leaf plots** would serve nicely since we are trying to compare the distributions of these data values. This way, the distributions are side-by-side and they can be ascertained accordingly.
(B) **Scatterplot of B *versus* A** would not serve this purpose in the least. As a matter of fact, it would be impossible to create because in order to do that you require <u>bivariate</u> data (two variables taken from each experimental unit). We haven't done that here because the only variable we collected from each person is their salary.
(C) **Parallel boxplots of A and B** would serve the same purpose as the back to back stem and leaf plots because it would show the distributions of the salaries side by side.
(D) **Histograms of A and B that are drawn to the same scale** should also serve this purpose because you can see the shape, center, and spread of each group side by side. Also, it's very important that you use the same scale for each because if they're not on the same scale, it can be misleading.
(E) **Dotplots of A and B that are drawn on the same scale** could also be useful because they can show the distributions of each variable side by side as well.

I think this is an important question because you may very well be asked to draw the 'appropriate' display for some data and you have to know what to do. If you want to <u>compare</u> distributions, everything but a scatterplot will get that done. A scatterplot can show a <u>relationship</u> between to variables (each garnered from the same experimental unit).

**21. Answer: (A)**

When reading these outputs, you have to know how to acquire the least squares regression line. Ignore all the stuff at the top (Regression, Residual, etc). Also, ignore R squared (adjusted) as this does not pertain to you. Here are the only parts of the output you need for this question:

| Variable | Coefficient | s.e. of Coeff | t-ratio | prob |
|---|---|---|---|---|
| Constant | 11.6559 | 0.3153 | 37 | $\le 0.0001$ |
| Size | 3.47812 | 0.294 | 11.8 | $\le 0.0001$ |

The only confidence interval you will ever be asked to create for regression is one for the slope of the least squares regression line. Recall, though, that degrees of freedom for regression are $n - 2 = 22 - 2 = 20$ (which is conveniently given to you in this example). You must tehn look up the critical value for 95% confidence and 20 degrees of freedom:

| df | | | | | |
|---|---|---|---|---|---|
| 20 | 1.325 | 1.725 | 2.086 | 2.197 | 2.528 |
| : | : | : | : | : | : |
| | 80% | 90% | 95% | 96% | 98% |

Confidence Level *C*

The formula for the CI is as follows:

$$b_1 \pm t^*_{n-2} \times SE_{b_1}$$

$$3.47812 \pm 2.086 \times 0.294$$

$$(2.864836,\ 4.091404)$$

Even though the actual confidence interval is not in the question, it's good to know what it means. We are 95% confident that for each additional kilobyte of size, the document requires somewhere between an additional 2.864836 minutes and 4.091404 minutes to print.

**22. Answer: (D)**

This question harps at the distinction between an observational study and a well-designed experiment. Since there was no random assignment of subjects to treatment groups, this is not an experiment. It is just an observational study. It's actually a retrospective study because past data was collected to acquire this information. Recall that the main difference between an observational study and an experiment is that an experiment can determine causality (provided that it's well-designed) and an observational study can only determine that a relationship exists between the variables. So…

(A) is TRUE → This should be included in the report because even though seatbelts were probably a factor, they probably weren't the only factor. This study could be confounded by driver behavior. That is, it may be hard to tell if the injury (or lack thereof) was because of the seatbelt or because of the driver's behavior.

(B) is TRUE → This should be included in the report because, again, you cannot tell if the injury was due to the seatbelt or the child's location in the car. Studies have shown that children who sit in the front of a car are more apt to be injured than children who sit in the back seat. So this, too, is a confounding factor.

(C) is TRUE → This should be included in the report because this was not an experiment. We didn't take kids in throw them into a car (some with seatbelts on, some without), crash the car into a pole, and record how things panned out. Since we cannot do this (for ethical reasons… hello?) a cause and effect relationship cannot be determined even if it does not exist.
(D) is FALSE → This should not be included in the report because in an observational study, we can only determine that in accidents where seatbelts were worn, less children were injured.
(E) is TRUE → This should be included in the report because one of the main caveats of experimental design is replication. If something is true, it should always be true, not just in this example.

**23. Answer: (E)**

In probability theory, we have three main types of events:

1. **Mutually Exclusive (a.k.a. disjoint) Events**: Outcomes can not happen at the same time.
2. **Independent Events**: Outcomes can happen at the same time, they just don't affect each other.
3. **Neither Independent nor Disjoint (a.k.a. dependent) Events**: Outcomes can happen at the same time and they do affect each other.

(A) is FALSE → If the events are mutually exclusive, they cannot be independent. They cannot exist in both categories at once.
(B) is FALSE → **I**f the events are independent, they cannot be mutually exclusive. They cannot exist in both categories at once.
(C) is FALSE → If the events are not mutually exclusive, they could be either independent or dependent.
(D) is FALSE → If the events are not independent, they could still be either mutually exclusive or dependent.
(E) is TRUE → These events cannot exist in both categories at once. If the events are categorized as mutually exclusive, they cannot be independent.

**24. Answer: (D)**

Here, the results of a statistical test and subsequent p-value are given and you are asked to make assertions based on this information:

Remember that in order for a test to be "statistically significant", the p-value must be less than the alpha level. Since the p-value is 0.24, you do not reject Ho and thus the results are not statistically significant at any reasonable alpha level [0.01, 0.05, or 0.10].

(A) is FALSE → In order for these data to be statistically significant, the p-value must be less than the alpha level.
(B) is FALSE → You need not do anything to the p-value. It is what it is. Leave it alone.
(C) is FALSE → Once again, you don't need to double the p-value unless you're given a one-tailed test and you need to convert it to a two-tailed test. In no way is this indicated here.
(D) is TRUE → This is the definition of a p-value. It is the probability of viewing data as extreme or more extreme than that which you saw, supposing the null hypothesis were correct.
(E) is FALSE → Even though it is the correct definition of what a p-value is, there is no reason to subtract that value from one.

**25. Answer: (E)**

This question deals with the appropriate design of a study. Make sure that you understand the different components that deal with the design and the implementation of a study as well as the different biases that could arise from your study.

(A) is FALSE → You cannot simply let subjects pick whichever drug they do/don't take.
(B) is FALSE → When you design a study, you must always make it to where the subjects are randomly assigned to treatments.
(C) is FALSE → Even though it is a paired design, you still need to utilize randomization. That is, you still need to randomly assign which group gets the old drug first and which gets the new drug first.
(D) is FALSE → The question states that they want to compare the old drug to the new drug. If you don't use the old drug, all you have evidence for is the idea that the drug works better than nothing. Bang-up job, genius.
(E) is TRUE → The subjects are randomly assigned to two groups and one group gets the old drug, the other gets the new drug. Perfect.

**26. Answer: (D)**

26. This is a question about sample size determination. It’s likely that this question (or one like it) will come up on the exam because this is one of the formulas that you have to memorize. The formulas for sample size determination depend upon the data:

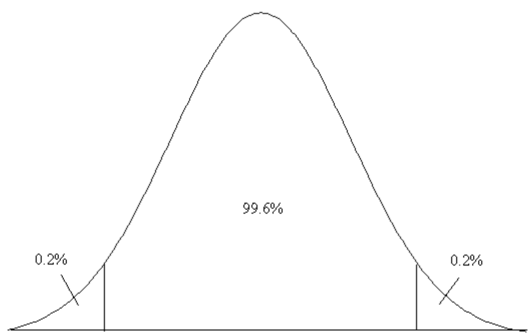
$$\text{Quantitative} \rightarrow \qquad n = \left(\frac{z \times \sigma}{m}\right)^2$$

$$\text{Categorical} \rightarrow \qquad n = \left(\frac{z}{m}\right)^2 p(1-p)$$

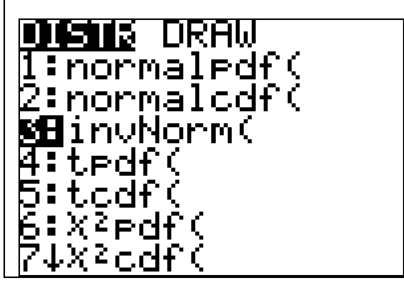
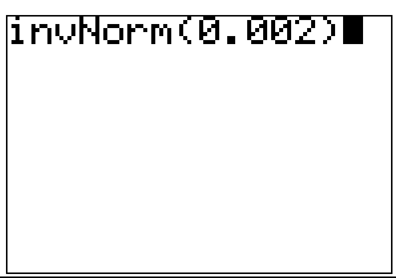
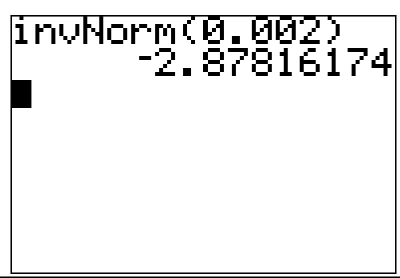
In the formula, the critical $z$ comes from the confidence level and the margin of error ($m$) will be in the question noted as either (a) “margin of error” or (b) it will follow the word “within”.

Also, for the formula that deals with categorical data, if there is no estimate for $p$ (or if the question asks for the “most conservative estimate”), use 0.5.

Because the question asks for a critical $z$ value that is not on the t-table (99.6% confidence), you have to get it yourself using the calculator:




Now use the calculator to get the critical value:

| 1. Press 2nd → VARS → 3:invNorm( | 2. Type invNorm(area to left) | 3. The value will inevitably be negative, so make it positive. |
|---|---|---|
|  |  |  |

Now just use the quantitative formula because this is quantitative data (it's a machine measuring the weights of snack foods):

$$n = \left(\frac{z \times \sigma}{m}\right)^2 = \left(\frac{2.878 \times 0.30}{0.12}\right)^2 = 51.76$$

Always round up! → $n = 52$

Note: The critical $z$ value for a confidence level very close to 99.6% is on the $t$ table (99.5% confidence) as $z = 2.807$. If you use this number in the formula, you get a very similar answer:

$$n = \left(\frac{z \times \sigma}{m}\right)^2 = \left(\frac{2.807 \times 0.30}{0.12}\right)^2 = 49.24$$

Among the choices, $n = 52$ is the 'minimum' sample size that would achieve his goals.

**27. Answer: (A)**

Be careful with these questions. This is a cumulative frequency histogram, not just a histogram. For example, look at the test score of 70. The cumulative frequency is about 25% which means that about 25% of the students scored a 70 on the test or less. Knowing this…

(A) is TRUE → The variability of the data can be determined by looking at the horizontal distance covered before achieving that percentile. I would guess that the 20$^{th}$ percentile [lower 20%] is at a test score around 65. So the lower 20% of the test scores ranged from about 35 to about 65. The 80$^{th}$ percentile [upper 20%] is at a test score of about an 80. So the upper 20% of test scores are spread out from about 80 to 100. So, the range of the lower 20% test scores is approximately 65 – 35 = 30 and the range of the upper 20% of test scores is approximately 100 – 80 = 20. Since the lower 20% of test scores are more spread out, they have more variability.

(B) is FALSE → The median test score should be at the 50% mark on the cumulative percent. Look at 50 on the $y$-axis and trace the graph to the right and you will see the median score is somewhere between 70 and 80. Not less than 50.

(C) is FALSE → If you look at the test score of 80, the cumulative frequency for that value is 60%. This means that 60% of the students scored an 80 or less.

(D) is FALSE → If you look at the cumulative frequency for a test score of 70, the cumulative frequency is about 25%. This means that 25% of students scored a 70 or less and logically, 75% of the students scored a 70 or greater.

(E) is FALSE → The values which occurred the most frequently are those for which there were the biggest "jumps" in the data. There seems to be the biggest "jump" in cumulative frequency from the score of 80 to the score of 85. This means that these values occurred the most frequently.

**28. Answer: (E)**

Remember that in order for a linear model to be appropriate for a set of bivariate data, the residual plot must have random scatter above and below zero with no discernable patterns. The first residual plot (of the original variables, *x* and *y*) shows a clear curved pattern in the residuals. The second regression is then run on the transformed variables (the log of *x* and the log of *y* were both taken) and the residual plot for the transformed data looks much better. So…

(A) is FALSE → There is definitely a nonlinear relationship between *x* and *y* as seen by the curved residual plot and Regression I definitely does not yield a better fit.
(B) is FALSE → For the same reason as (A) … there is not a linear relationship between *x* and *y*.
(C) is FALSE → You cannot tell if the correlation is negative or positive based upon a residual plot.
(D) is FALSE → Even though there is a nonlinear relationship between *x* and *y*, Regression I is still not the better fit.
(E) is TRUE → There is a nonlinear relationship between *x* and *y* and Regression II yields the better fit.

**29. Answer: (D)**

Recall that a two-sided hypothesis test and a confidence interval with a complimentary alpha level will always yield consistent results. In this case, the confidence level is 98% and the alpha level is 0.02, so these results will be consistent.

(A) is FALSE → Even though the question doesn't indicate that we know the value of $\sigma$, the sample size of 500 renders that fact almost irrelevant. At a sample size of 500, *s* is an excellent estimate of $\sigma$.
(B) is FALSE → Again, the population need not necessarily be normally distributed if the sample size is 500. The sampling distribution is approximately normal regardless of that fact.
(C) is FALSE → You do not need the entire data set to get this question. All you need is the confidence interval and the hypothesis test and you're good to go.
(D) is TRUE → The value for the null hypothesis is 40,000. The 98% confidence interval is entirely above 40,000 so the null hypothesis would be rejected. We are 98% confident that the true mean income is not $40,000. Also, though not asked in this question, you can also conclude that the p-value is less than 0.02 since you reject $H_o$.
(E) is FALSE → The question says that the null hypothesis is not rejected. This is not correct. If the value from your hypothesis test is not in your confidence interval, then you must reject the null.

**30. Answer: (C)**

On your formula sheet, you are given the following information regarding the sampling distribution of the sample mean:

$$\mu_{\bar{x}} = \mu \text{ and } \sigma_{\bar{x}} = \sigma / \sqrt{n}$$

Since we are taking samples of size 2 from this (very small) population, we know the following about the sampling distribution of $\bar{x}$:

$$\mu_{\bar{x}} = \mu = 4.25$$

$$\sigma_{\bar{x}} = \sigma / \sqrt{n} = 1.92 / \sqrt{2} = 1.358$$

So… $\mu_{\bar{x}} = 4.25$ and $\sigma_{\bar{x}} < 1.92$

**31. Answer: (B)**

Since I've gone over this one a bunch of times in previous examples, we'll just go over the **right** answer:

Recall that the slope is defined as follows:

"We predict, on average, that for each additional increase in *x*, *y* will increase by [enter slope here]"

Since *x* is the number of degrees Fahrenheit by which the temperature exceeds $50^{o}$ and *y* is the number of chirps per minute, the slope must mean that for each additional $1^{o}$ increase in temperature, the number of chirps per minute increases by 3.41. [Remember that 10.53 is not the slope. It's the *y*-intercept].

**32. Answer: (E)**

This example follows a binomial distribution. Recall the 4 qualifications for a binomial distribution:

1) There are two outcomes (success and failure)
2) The probability of success is constant
3) The trials are independent
4) There are a fixed number of trials

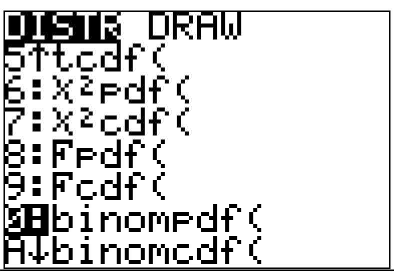
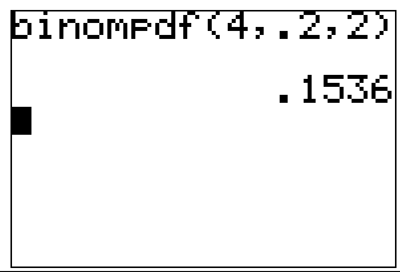
This example works nicely:

1) There are 2 outcomes [you either win the prize or you don't]
2) The probability of success is constant [It's always 0.20 because there are 5 boxes and one prize]
3) The trials are independent [If you win this time, it has no bearing on you winning the next time]
4) There are a fixed number of trials [You are guessing 4 times]

Since the question is asking for the probability of getting <u>exactly</u> *k* successes within *n* trials (in this case, exactly 2 correct out of 4 guesses), you use the binomial formula given to you on your formula sheet:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} = \binom{4}{2} (0.2)^2 (0.8)^2$$

This question just wants you to use the formula with the numbers in the appropriate places. If you had to evaluate this, though, you can do it in the calculator:

| 1. Press [2nd] → [VARS] → 0:binompdf( | 2. Type binompdf (*n*, *p*, *k*) | 3. This gives you the probability of getting <u>exactly</u> 2 successes out of 4 trials if the probability of success is 0.20. |
|---|---|---|
|  |  |  |

(A) $\frac{2!}{5!}$ → This gives the number of possible ways in which you can get 2 successes out of 5 trials. It is called a permutation. This is <u>not</u> covered in this class

(B) $\frac{(0.2)^2}{(0.8)^2}$ → This is a crap answer. Did you fall for this? Don't fall for this.

(C) 2(0.2)(0.8) → This would follow a binomial distribution if there were 2 ways the event could occur.

(D) $(0.2)^2(0.8)^2$ → This would have been the solution if the question had asked: "What is the probability of guessing correctly on the first two attempts, but then guessing incorrectly on the last two".

**33. Answer: (B)**

In this question, you are kind of tricked because the sample size is small, so you think that inference cannot be done. However, the question states that dotplots of the data indicate that the assumption of normality is not unreasonable, so the small sample size is not important. Remember that you use a *t*-distribution to estimate the *z*-distribution when the population standard deviation is unknown. So…

(A) is FALSE → We <u>always</u> construct confidence intervals using our statistics in hopes of capturing parameters. In this case, we are using the sample mean carbon content to give us insight into what the population mean carbon content could be.
(B) is TRUE → The question states that "there are no historical data on the variability of the process". This means that the population standard deviation is unknown so we use the sample standard deviation to estimate it. I know the answer says that the sample variance is used as an estimate of the population variance, but remember:

$$\text{Variance} = \text{Standard Deviation}^2$$

If the population standard deviation is unknown, then so is the variance.

(C) is FALSE → We should always use data rather than theory to judge normality.
(D) is FALSE → You can use data from however many days you think it takes to get a representative sample. In no way does this mean that you have to use a *t* distribution.
(E) is FALSE → It is good to use a *t*-distribution when the sample size is small, but the main reason is because the standard deviation for the population is unknown. You can use a *z*-interval with a small sample size if the population standard deviation is known and the population from which the data comes is approximately normal. So the statement, "A *z*-interval should never be used with a small sample" is false.

**34. Answer: (C)**

This example uses bivariate data because two measures were recorded for each rat: the amount of caffeine consumed ($x$) and the rat's blood pressure ($y$). The correlation is calculated as 0.428. This is $r$. This means is there is a moderately weak positive correlation between the amount of caffeine consumed by a rat and that rat's blood pressure. So…

(A) is FALSE → The correlation for our sample is 0.428. This is not the correlation for the entire population. Although, if these 100 rats were randomly sampled from the population, the population correlation should be close to that.
(B) is FALSE → Correlation does not imply causation. In no way will not drinking the caffeine cause a reduction in blood pressure. There could be a multitude of confounding variables that we do not know about.
(C) is TRUE → It is the coefficient of determination that describes the percent of variability in y explained by $x$. In this example:

$$r^2 = 0.428^2 = 0.183$$

This means that about 18% of the variability in a rat's blood pressure can be explained by the amount of caffeine the rat consumed. Another way of saying this (you don't have to know this for free response questions, but it could be on the multiple choice section). About 18% of the variation in blood pressure can be explained by the linear association between blood pressure and caffeine consumed. This is equally as viable an answer.

(D) is FALSE → We don't know what the rats like. Rats don't talk. This is stupid.

(E) is FALSE → A weak correlation does not necessarily mean that the relationship is nonlinear. This could be the case because nonlinear relationships weaken the correlation, but it could just be a sort of weak linear association.

**35. Answer: (C)**

Remember that the power of a test [known as $(1-\beta)$] is not something that you have to calculate; just something you have to know. The power of a test is the probability that you correctly reject a false null hypothesis (it's your BS detector). Recall that to increase the power, the two ways to do that are to increase the sample size and/or increase the effect size. Also, remember:

$$\alpha \uparrow \Rightarrow \beta \downarrow \Rightarrow (1-\beta) \uparrow$$

So, to achieve the highest power, you just have to find the highest alpha level (Type I Error Rate) and the highest sample size will give you the most power.

**36. Answer: (B)**

Since the balls are being replaced, you can easily know all possible combinations of balls that can be drawn and thus find the average of each:

| Balls Drawn | $\overline{X}$ |
|---|---|
| 1 and 1 | 1 |
| 1 and 2 | 1.5 |
| 1 and 3 | 2 |
| 2 and 1 | 1.5 |
| 2 and 2 | 2 |
| 2 and 3 | 2.5 |
| 3 and 1 | 2 |

| | |
|---|---|
| 3 and 2 | 2.5 |
| 3 and 3 | 3 |

Now there are 9 possibilities, you know what the sampling distribution of $\overline{X}$ is just by looking at how often each $\overline{X}$ occurs:

| $\overline{X}$ | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|
| Probability | $\frac{1}{9}$ | $\frac{2}{9}$ | $\frac{3}{9}=\frac{1}{3}$ | $\frac{2}{9}$ | $\frac{1}{9}$ |

**37. Answer: (E)**

Since the simple random sample yields an $\bar{x}$ of 15 $\pm$ 3, this gives a resulting confidence interval of (12, 18). This can be viewed from two perspectives:

(1) We are 95% confident that $\mu$ lies between 12 and 18.

(2) In all intervals constructed the same way (of the same sample size from the same population), we expect 95% of them to capture $\mu$.

So…

(A) is FALSE → To discuss "95% of population measurements" is a stupid statement. We have never covered "population measurements".

(B) is FALSE → This is because "95 of sample measurements" will look like whatever the hell they want to look like. However, if we create intervals around these 'sample measurements', 95% of these intervals should capture $\mu$. However, our confidence interval is not the benchmark for all confidence intervals.

(C) is FALSE → This is for the same reason as choice (B). Again, our confidence interval is not the benchmark for all confidence intervals. It's close to the right answer, but it's actually incorrect and that's why most students chose it. Had it said "In 100 intervals constructed the same way, we expect 95 of them to capture $\mu$", that would have been correct.

(D) is FALSE → This is because $P(12 \le \bar{x} \le 18)$ = 1. The sample average is <u>always</u> directly in the center of any confidence interval.

(E) is TRUE → This is because in all intervals constructed the same way, we expect 95% of them to capture $\mu$. So if $\mu$=19, we would be kind of surprised. We are 95% confident that $\mu$ is between 12 and 18. Only 5% of all 95% confidence intervals will not contain $\mu$, and it looks like we're in that 5% of intervals. So if $\mu$ = 19, this $\bar{x}$ of 15 would be unlikely.

**38. Answer: (E)**

Since this question is asking for the answer in terms of those who are <u>against</u> the tax increase, do it entirely from that perspective ($p = 0.35$):

This is probably the way that these questions will be asked in that they will use a rather large sample size ($n = 500$) as sort of a hint that you should use a normal model. Since there are at least 10 successes and at least 10 failures:

$$np = (500)(0.35) = 175 \geq 10 \text{ and } n(1-p) = (500)(0.65) = 325 \geq 10$$

…a normal model is appropriate. This makes these two choices incorrect:

(A) $\binom{500}{200}(0.65)^{200}(0.35)^{300} \rightarrow$ This is the probability of getting <u>exactly</u> 200 people who are <u>in favor of</u> the taxes (because it uses 0.65 first) in the sample of 500 people.

(B) $\binom{500}{200}(0.35)^{200}(0.65)^{300} \rightarrow$ This is the probability of getting <u>exactly</u> 200 people who are <u>against</u> the taxes (because it uses 0.35 first) in the sample of 500 people.

Now that you have a normal model, you need a mean and a standard deviation. These are given to you on your formula sheet:

$$\mu_{\hat{p}} = p = 0.65$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.35)(0.65)}{500}}$$

The question asks for the probability that more than 200 people are against the taxes. This is the same as asking for the probability that more than $200/500 = 0.40$ are against the taxes. Now, just convert all the information into a $z$-score and find the probability of getting a $z$-score <u>bigger</u> than that:

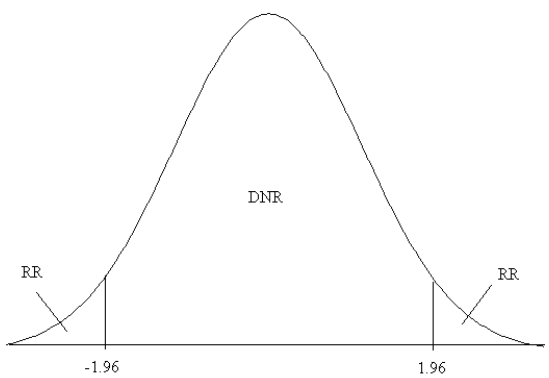
$$z = \frac{x-\mu}{\sigma} = \frac{0.40 - 0.35}{\sqrt{\frac{(0.35)(0.65)}{500}}}$$

**39. Answer: (A)**

The only way that a two-sided test can find significance (a.k.a. reject $H_0$) and a one-sided test does not is if the researcher chose the wrong direction for the one-sided alternative. Let's discuss this from the perspective or the rejection region(s):

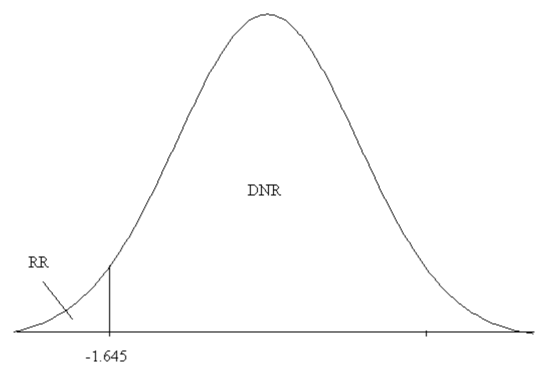
<u>Sally</u>

Results were significant (rejected $H_0$)
Used two-sided test

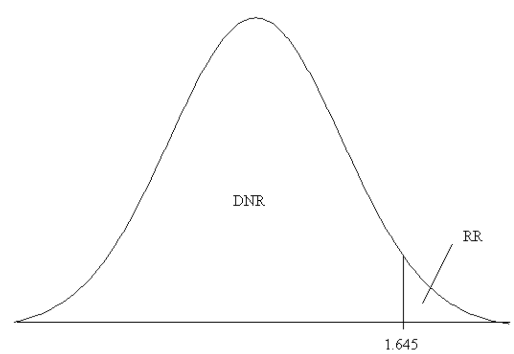



<u>Betty</u>

Results were <u>not</u> significant (did <u>not</u> reject $H_0$)
Used one-sided test (direction not known)




OR




The toughest part of this question is its ambiguity. Betty could have chosen the 'wrong' direction in her hypothesis test. Therefore, just check to see if any of the choices for a test statistic fall in Sally's rejection region and doesn't fall into <u>either</u> of Betty's (since you don't know which one it was). The only answer choice that satisfies this is (A) –1.980.

**40. Answer: (B)**

Here, the person is attempting to do a two sample *t*-interval:

(A) is FALSE → The two sample sizes do not need to be equal in order to perform any inference on independent means.
(B) is TRUE → The student gathered the ages of every prime minister and every president, so a confidence interval is unnecessary. He has the parameters (the true mean ages of both populations). He's a winner. Game over.
(C) is FALSE → The distribution of ages could very well not be similar and the test can still be performed. They just both have to be large samples or from normal populations in order to be valid.
(D) is FALSE → We don't really know what the shapes of the ages would be. They're probably all centered around late 50's with a fairly normal distribution, though you cannot really speculate to this effect without the data.
(E) is FALSE → This is for the same reasons as choice (D). You have no idea what the distributions looks like and cannot make assumptions to that effect.