2003 AB Section I Part A   No Calculator

1) $y=(x^3+1)^2$

$\frac{dy}{dx}=2(x^3+1)(3x^2)$

$=6x^2(2x^3+1)$ E

2) $\int_0^1 e^{-4x}dx$

$u=-4x$

$du=-4dx$

$-\frac{1}{4}\int e^u du = -\frac{1}{4}e^{-4x}\Big|_0^1$

$=-\frac{1}{4}e^{-4}-(-\frac{1}{4})$

$=\frac{-e^{-4}}{4}+\frac{1}{4}$ D

3) $\lim_{x\to\infty}=2$ E

4) $y=\frac{2x+3}{3x+2}$

$\frac{dy}{dx}=\frac{(3x+2)(2)-(2x+3)(3)}{(3x+2)^2}$

$=\frac{6x+4-6x-9}{(3x+2)^2}=\frac{-5}{(3x+2)^2}$ D

$\int_0^{\pi/4}\sin x\,dx=$

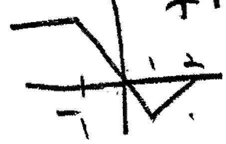
$-\cos x\Big|_0^{\pi/4}=$

$-\cos\pi/4-(-\cos 0)=$

$-\sqrt{2}/2+1$ D

$\lim_{x\to\infty}=1/4$ C

7) $f'(x)$ B



8) $\int x^2\cos(x^3)dx$ $u=x^3$

$du=3x^2dx$

$\frac{1}{3}\int\cos u\,du$

$\frac{1}{3}\sin(x^3)+C$ B

9) $f(x)=\ln(x+4+e^{-3x})$

$f'(x)=\frac{1-3e^{-3x}}{x+4+e^{-3x}}$

$f'(0)=-2/5$ A

10) decreasing / concave down B

11) $u=2x+1$ $du=2$ $\int_0^2 \to \frac{1}{2}\int_1^5 u^{1/2}=\frac{1}{2}\int_1^5\sqrt{u}\,du$

when $x=2$ $u=5$

when $x=0$ $u=1$

C

12) $\frac{dV}{dt}=k\sqrt{V}$ E

rate of change

13) A (can't take derivative at sharp point)

14) $y=x^2\sin 2x$

$\frac{dy}{dx}=x^2\cos 2x(2)+\sin 2x(2x)$

$2x(x\cos 2x+\sin 2x)$ E

15) $f'(x) = x^2 - \frac{2}{x}$
f(x) dec. where f'(x) is neg.
$\frac{x^3-2}{x} = 0$

$x^3 = 2$
$x = \sqrt[3]{2}$ $x = 0$

$f'(x)$ $+$ $-$ $+$ at 0, $\sqrt[3]{2}$

f(x) dec $(0, \sqrt[3]{2})$ D

16) $f'(1) = \frac{-2-7}{-2-1} = \frac{-9}{-3} = 3$ C

17) $f(x) = 2xe^x$
$f'(x) = 2xe^x + 2e^x$
$f'(x) = 2e^x(x+1)$
$f''(x) = 2e^x + (x+1)2e^x$
$2e^x(1+x+1)$
$2e^x(x+2) = 0$ $x = -2$

$f''(x)$ $-$ $+$ at -2

f c.c down $x < -2$ A

18) g decreases when g' is negative
$-2 \leq x \leq 2$ A

19) Slope $= (2x+3)$ $(1,2)$
only B, C have slopes of $2x+3$
only C satisfies the point $(1,2)$ C

20) I $\lim\limits_{x \to 3^+} = \lim\limits_{x \to 3^-}$ so $\lim\limits_{x \to 3}$ exists ✓

II since $f(3) = 5$ f is continuous at 3 ✓

III since $f' = 1$ for $x < 3$, $f' = 4$ for $x > 3$, not diff at 3 ✗
D

21) POI where f'(x) changes from ±
a, 0 A

22) $y' = -6x + 6$

$y = \int -6x + 6$
$y = -3x^2 + 6x + C$
$y(0) = -3(0) + 6(0) + C = 5$
$C = 5$
$y = -3x^2 + 6x + 5$
$y(1) = -3(1) + 6(1) + 5$
$= -3 + 6 + 5 = 8$ D

23) $\frac{d}{dx}\left[\int_0^{x^2} \sin(t^3)\,dt\right]$

$\sin(x^2)^3 \, (2x)$
$\sin x^6 \, (2x)$ E

24) $f(x) = 4x^3 - 5x + 3$
$f'(x) = 12x^2 - 5$
$f'(-1) = 7$ $(-1, 4)$
$y - 4 = 7(x+1)$
$y - 4 = 7x + 7$
$y = 7x + 11$ C

25) $x(t) = 2t^3 - 21t^2 + 72t - 53$

$v(t) = 6t^2 - 42t + 72 = 0$

$6(t^2 - 7t + 12) = 0$

$(t-4)(t-3) = 0$

$t = 4, 3$ E

26) $3y^2 - 2x^2 = 6 - 2xy$ $(3,2)$

$6y\frac{dy}{dx} - 4x = -2x\frac{dy}{dx} + y(-2)$

$6y\frac{dy}{dx} + 2x\frac{dy}{dx} = -2y + 4x$

$(6y + 2x)\frac{dy}{dx} = -2y + 4x$

$\frac{dy}{dx} = \frac{-2y + 4x}{6y + 2x}$

$dy/dx = \frac{-4+12}{12+6} = \frac{8}{18} = \frac{4}{9}$ B

* 27) $f(x) = x^3 + x$

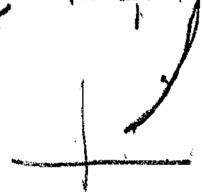
$g(2) = f^{-1}(2) = 1$

B

28) g increasing, concave up

$g(4) = 12$ $g(5) = 18$ $m = \frac{6}{1}$

$g(6) = ?$



A, B too small

slopes are increasing

since CCU

(6, 21) (5, 18) $m = 3$ to small

(6, 24) (5, 18) $m = 6$ to small

(6, 27) (5, 18) $m = 9$ E

2003 Multiple Choice - Calculator

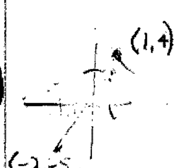
$\star$ must use calculator

$\star$76) $v(t) = 3 + 4.1\cos(.9t)$
$a(4) =$ $\boxed{\text{math } 8}$ nderiv$(3+4.1\cos(.9t), x, 4) = 1.633$ $\boxed{C}$

77) $\int_{-3}^{3}(f(x)+1)dx = \int_{-3}^{3} f(x)dx + \int_{-3}^{3} 1\,dx = -2+2-2 + x\Big|_{-3}^{3} = -2 + (3-(-3)) = 4$ $\boxed{C}$

78) $\frac{dr}{dt} = .2$ $\quad \frac{dA}{dt} = ?$ $\quad C = 20\pi$
$A = \pi r^2$ $\quad 2\pi r = 20\pi$
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ $\quad r = 10$
$\frac{dA}{dt} = 2\pi(10).2$
$\frac{dA}{dt} = 4\pi$ $\boxed{C}$

79) I - Yes $\quad$ II. Yes $\quad$ III No $\quad$ $\boxed{D}$

80) 

A. True crosses x-axis $\quad$ B. False not necessarily $\quad$ C. True must pass through y=3 $\quad$ D. True $\frac{4-(-5)}{1-(-2)} = \frac{9}{3} = 3$ $\quad$ E. True there's a max $\quad$ $\boxed{B}$

$\star$81) $f'(x) = \sin(x^2+1)$ $\quad (2,4)$
find critical #'s for $f'(x)$ from $(2,4)$ look at graph
$f'(x) = 0$ 4 times $\boxed{D}$

$\star$82) $r(t) = t^3 - 4t^2 + 6$ $\quad 0 \le t \le 8$
want change in altitude, $r(t)$ is a rate, so need

$\int r(t)dt$ (eliminates D, E)

altitude decreases when rate altitude changes < 0 $\quad t = 1.512, 3.514$
$\int_{1.512}^{3.514} r(t)dt$ $\boxed{A}$

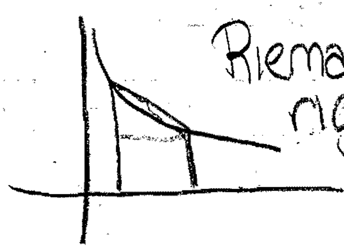
$\star$83) $v(t) = e^t + te^t$ $\quad (0,3)$
average velocity $\frac{1}{3}\int_0^3 (e^t + te^t)\,dt = 20.086$ $\boxed{A}$

★ 84) $\frac{dt}{dP} = -110e^{-.4t}$ $(0, 75°)$

$-110\int_0^5 e^{-.4t}$ $u = -.4t$, $du = -.4$

$-110(-\frac{1}{.4})\int_0^5 e^u du \rightarrow \int_0^5 275e^{-.4t} dt$

$350 - 237.783 = 112.°F$ $\boxed{A}$

85) 

Riemann sum overestimates, right sum underestimates $\boxed{A}$

★ 86) y-axis $y = \tan^{-1}x$

$y = 3$, $x = 1$

$\int_0^1 (3 - \tan^{-1}x)^2 dx = 6.612$ $\boxed{B}$

★ 87) $f'(x) = \frac{\sqrt{x}}{1+x+x^3}$

inflection pt where $f'(x)$ changes from inc/dec or dec/inc $\rightarrow$ $.0473$ $\boxed{B}$

88) $\frac{1}{4-2}\int_2^4 f(t)dt = 1$

$\int_2^4 f(t)dt = 2$

area from $(2,4) = 2$ $\boxed{C}$

89) $f(2) = 3$ $f'(2) = -5$

$(2,3)$ $m = -5$ $g(x) = xf(x)$

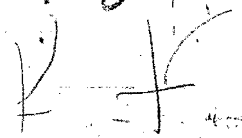
$g'(x) = xf'(x) + f(x)(1)$ $g(2) = 2(f(2))$

$g'(2) = 2(-5) + 3(1)$ $g(2) = 2(3) = 6$

$g'(2) = -10 + 3 = -7$ $g(2) = 6$ $g'(2) = -7$

$y - 6 = -7(x - 2)$ $\boxed{D}$

90) $\frac{f(b)-f(a)}{b-a}$ must be pos



$\boxed{B}$

91) $a(t) = \ln(1+2^t)$

★ 92) $g(x) = \int_{-1}^{3} \sin(t^2)dt$

g decreases where $g'(x)$ is negative

$g'(x) = \sin(t^2)$ look at graph

$1.772 \leq x \leq 2.507$ $\boxed{D}$