2008 APCalculus AB

no calculator

1) $\lim_{x\to\infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$

$= \frac{6x-2x^2-3+x}{x^2+2x-3}$

$= \frac{-2x^2+7x-3}{x^2+2x-3}$

$\lim_{x\to\infty} \leftrightarrow$ horiz asy.
compare degree
of top and bottom:
Same $\to \boxed{-2}$ B

2) $\int \frac{1}{x^2}\,dx = \int x^{-2}$

$= \boxed{-x^{-1}+C}$ D

3) $f(x)=(x-1)(x^2+2)^3$
product/chain rule
$f'(x) = (x-1)(3)(x^2+2)^2(2x) + (x^2+2)^3$

$f'(x)= (x^2+2)^2\left[6x(x-1)+x^2+2\right]$

$=\boxed{(x^2+2)^2\left[7x^2-6x+2\right]}$ D

4) $\int(\sin(2x)+\cos(2x))\,dx$

$\int \sin(2x) + \int \cos(2x)$

$u=2x$ $du=2$ $u=2x$ $du=2$

$\frac{1}{2}\int \sin u + \frac{1}{2}\int \cos u$

$\boxed{-\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C}$ B

5) $\lim_{x\to 0} \frac{5x^4+8x^2}{3x^4-16x^2} = \frac{0}{0}$

AB
or factor and plug

L'hopitals Rule
BC

$\frac{f'(x)}{g'(x)}$

$\frac{f'(x)\ 20x^3+16x}{g'(x)\ 12x^3-32x}$ (still gives $\frac{0}{0}$)

L'hopitals again

$\lim_{x\to 0} \frac{60x^2+16}{36x^2-32} = \frac{16}{-32} = \boxed{\frac{-1}{2}}$ A

6) $\boxed{\text{I}}$ $\lim_{x\to 2^+} = \lim_{x\to 2^-} = \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} = 4$

II not continous at $x=2$
since $x=2$ is a VA

III not differentiable at $x=2$,
since $x=2$ is a VA A

7) $v(t)=3t^2+6t$

$x(t)=\int 3t^2+6t$

$x(t)= t^3+3t^2+C$

$x(0)=0+0+C=2$ $C=2$

$x(t)=t^3+3t^2+2$

$x(1)=1+3+2=\boxed{6}$ B

8) $f(x)=\cos(3x)$

$f'(x)=-\sin(3x)(3)$

$f'(x)=-3\sin(3x)$

$f'(\pi/9)=-3\sin(3\cdot\frac{\pi}{9})$

$=-3\sin(\pi/3)$

$=-3(\sqrt{3}/2)$

$=\boxed{\frac{-3\sqrt{3}}{2}}$ E

9) $g(x) = \int_{-2}^{x} f(t)\,dt$

$g(-3) = \int_{-2}^{-3} f(t) = \frac{1}{2}(1)(3)$
$= 3/2$

$g(-2) = \int_{-2}^{-2} f(t) = 0$

$g(0) = \int_{-2}^{0} f(t) = \frac{1}{2}(2)(3) = 3$

$g(1) = \int_{-2}^{1} f(t) = 3 + \frac{1}{2}(1)(2)$
$= \boxed{4}$

$g(2) = \int_{-2}^{3} f(t) = 4 - 1 = 3$

D

10) A is the exact area
B is an over estimate
C is an underestimate *
D is greater than a right sum
E is greater than a right sum

C

11) B (look for where f is increasing → f' is pos; where f is decreasing f' is negative)

12) $f(x) = e^{(2x^{-1})}$
$f'(x) = e^{(2x^{-1})}(-2x^{-2})$
$= \boxed{\frac{-2e^{\frac{2}{x}}}{x^2}}$

D

13) $f(x) = x^2 + 2x$
$f(\ln x) = (\ln x)^2 + 2\ln x$
$\frac{d}{dx}\left[(\ln x)^2 + 2\ln x\right] =$
$= 2(\ln x)\left(\frac{1}{x}\right) + 2\left(\frac{1}{x}\right)$
$= \frac{2\ln x}{x} + \frac{2}{x} = \boxed{\frac{2\ln x + 2}{x}}$

A

14) E since f'' changes from + to − in between 0 and 2, there must be a point where f changes concavity. Be careful of choice D (there could be other x's where f''(x)=0 in between 0 and 2)

TRICKY QUESTION!

15) $\int \frac{x}{x^2-4}\,dx$ $\quad u = x^2 - 4$
$\quad du = 2x\,dx$

$\frac{1}{2}\int \frac{2x}{x^2-4}\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\boxed{\frac{1}{2}\ln|x^2-4| + C}$ C

16) $\sin(xy) = x$

$\cos(xy)\left(x\boxed{\frac{dy}{dx}} + y\right) = 1$

$x\frac{dy}{dx} + y = \frac{1}{\cos(xy)}$

$x\frac{dy}{dx} = \frac{1}{\cos(xy)} - y$

$x\frac{dy}{dx} = \frac{1 - y(\cos xy)}{\cos(xy)}$

$\boxed{\frac{dy}{dx} = \frac{1 - y(\cos xy)}{x\cos(xy)}}$

D

17) $g(x) = \int_0^x f(t)\,dt$

$f(t) = g'(x)$

g has point of inflection where $f''(x)$ changes from + to −, and $f'(x)$ changes from inc to dec (or dec to inc)

$\boxed{x = 2,5}$ C

18) $x + y = k$ tangent have same slope

$y = -x + k$ $y = x^2 + 3x + 1$

$y' = -1$ $y' = 2x + 3$

$2x + 3 = -1$ (set slopes =)

$2x = -4$ $x = -2$ (plug in x to find y)

$y = -1$ (solve for k)

$-2 + (-1) = k$

$\boxed{-3 = k}$ A

19) $y = \frac{5 + 2^x}{1 - 2^x}$

this question was not counted in the final scoring of the 2008 exam. Extremely TRICKY!

$\lim_{x\to\infty}$ and $\lim_{x\to-\infty}$ must be considered E

20) $f''(x) = x^2(x-3)(x-6)$

$x = 0, 3, 6$

$$\begin{array}{ccccccc} + & & + & & - & & + \\ \longleftarrow & 0 & \text{———} & 3 & \text{———} & 6 & \longrightarrow \end{array}$$

$\boxed{\text{POI at } x = 3, 6}$ D

21) velocity increases when acceleration is positive (so $x(t)$ is concave up)

$\boxed{0 < t < 2}$ A

22) $\boxed{\frac{dp}{dt} = k\,\boxed{p}\,(N - p)}$

proportional → multiply, B

if N is total # of people, and p is # who heard rumor

N−p # # who have not heard it.

k is a random constant.

23) $\frac{dy}{dx} = \frac{x^2}{y}$ $y(3)=-2$

$\int y\,dy = \int x^2\,dx$

$\frac{y^2}{2} = \frac{x^3}{3} + C$

$\frac{(-2)^2}{2} = \frac{3^3}{3} + C$

$2 = 9 + C$ $C = -7$

$\frac{y^2}{2} = \frac{x^3}{3} - 7$

$y^2 = \frac{2x^3}{3} - 14$

$\boxed{y = \pm\sqrt{\frac{2x^3}{3} - 14}}$

to decide + or −, plug in $y(3)=-2$. Only E works

24) $f(2)=1$ $f'(2)=4$

$(2,1)$ $m=4$

~~$f''(2)=3$~~

$y-1 = 4(x-2)$

$y-1 = 4x-8$

$y = 4x-7$ (tangent line)

$y = 4(1.9) - 7$

$7.6 - 7 = \boxed{.6}$ B

26) $y = \arctan(4x)$ $x = \frac{1}{4}$

$y' = \frac{u'}{1+u^2} = \frac{4}{1+(4x)^2} = \frac{4}{1+16x^2}$

$y'(1/4) = \boxed{2}$ A

27) C plug in points into each choice

*28) $f(3)=15$ $f(6)=3$

$f'(3)=-8$ $f'(6)=-2$

$g(x) = f^{-1}(x)$ find $g'(3)$

$\boxed{g'(x) = \frac{1}{f'(g(x))}} = \frac{1}{f'g(3)}$

Ⓐ

$g'(3) = \frac{1}{f'(g(3))} = \frac{1}{f'(6)} = \frac{1}{-2}$

since f is (6,3)
g is (3,6) (inverse)

rare formula - hardly anyone ever gets this right when on exam

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25) $f(x) = \begin{cases} cx+d & x \le 2 \\ x^2 - cx & x > 2 \end{cases}$ $\left.\begin{matrix} 2c+d \\ 4-2c \end{matrix}\right\}$ set = $2c+d = 4-2c$

$f'(x) = \begin{cases} c \\ 2x-c \end{cases}$ $c = 2(2) - c$

$2c = 4$

$\boxed{c=2}$ B

$2c+d = 4-2c$

$4+d = 0$ $\boxed{d=-4}$ $\boxed{c+d=-2}$