AB 2012 MC Solutions

1	$y = x\sin x \Rightarrow \dfrac{dy}{dx} = (1)(\sin x) + (x)(\cos x) = \sin x + x\cos x$ $\boxed{B}$
2	$f(x) = 300x - x^3 \Rightarrow$ increasing when $f'(x) \geq 0$ $f'(x) = 300 - 3x^2 = 3(10+x)(10-x) \geq 0$ when $-10 \leq x \leq 10$ or $[-10,10]$ $\boxed{B}$
3	$\int \sec x \tan x\, dx = \sec x + C$ $\left(\dfrac{d}{dx}\sec x = \sec x + \tan x\right)$ $\boxed{A}$
4	$f(x) = 7x - 3 + \ln x \Rightarrow f'(x) = 7 + \dfrac{1}{x}$ $f'(1) = 7 + \dfrac{1}{1} = 8$ $\boxed{E}$
5	$\lim_{x\to 4^-} f(x) = 2 \neq \lim_{x\to 4^+} f(x) = 4$ hence $\lim_{x\to 4} f(x)$ does not exist $\boxed{C}$
6	$v(t) = 6t - t^2 \geq 0$ on $[0,3]$ Total Distance $= \int_0^3 \lvert v(t)\rvert\, dt = \int_0^3 \lvert 6t - t^2\rvert\, dt = \int_0^3 (6t - t^2)\, dt = 3t^2 - \dfrac{t^3}{3}\Big\rvert_0^3 = 27 - 9 = 18$ $\boxed{D}$
7	$y = (x^3 - \cos x)^5 \Rightarrow y' = 5(x^3 - \cos x)^4 (3x^2 + \sin x)$ $\boxed{E}$
8	right Riemann sum $= 50 + \int_0^{15} R(t)\, dt = 50 + [3(6.2) + 5(5.9) + 3(5.6)] = 114.9$ liters $\boxed{C}$
9	$\lim_{x\to 2} \dfrac{(2x+1)(x-2)}{x-2} = \lim_{x\to 2}(2x+1) = 5 = k$ $\boxed{E}$

10	$y = e^{x/2} > 0$ on $[0,2]$ so area $= \int_0^2 e^{x/2}\, dx = 2e^{x/2}\Big]_0^2 = 2(e^1 - e^0) = 2e - 2$ $\boxed{A}$
11	$f(x) = \sqrt{\lvert x-2\rvert} \Rightarrow f(2) = 0$ so C and E are false. $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = 0$ so D is false and f is continuous $f(x) = \begin{cases} \sqrt{2-x}, & x < 2 \\ \sqrt{x-2}, & x \geq 2 \end{cases} \Rightarrow f'(x) = \begin{cases} -\tfrac{1}{2}(2-x)^{-1/2}, & x < 2 \\ \tfrac{1}{2}(x-2)^{-1/2}, & x > 2 \end{cases} \Rightarrow \lim_{x\to 2} f(x) = \varnothing$ so f not differentiable $\boxed{A}$
12	$u = \sqrt{x}$, $du = \dfrac{1}{2\sqrt{x}}dx \Rightarrow 2du = \dfrac{1}{\sqrt{x}}dx$ and when $x = 4, u = \sqrt{4} = 2$, when $x = 1, u = \sqrt{1} = 1$ $\int_1^4 \dfrac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2\int_1^2 e^u\, du$ $\boxed{C}$
13	$\int_1^5 f(x)dx = \int_1^3 2dx + \int_3^5 (x-1)dx = 2x\Big]_1^3 + \dfrac{x^2}{2} - x\Big]_3^5 = [6-2] + \left[\left(\dfrac{25}{2} - 5\right) - \left(\dfrac{9}{2} - 3\right)\right] = 4 + \dfrac{15}{2} - \dfrac{3}{2} = 10$ $\boxed{D}$
14	$\dfrac{d}{dx}[f(g(x))] = f'[g(x)]g'(x)$ so $\dfrac{d}{dx}[f(g(x))]_{x=3} = f'[g(3)]g'(3) = f'(7)g'(3) = \dfrac{7}{3\sqrt{5}}(3) = \dfrac{7}{\sqrt{5}}$ $\boxed{A}$ $f'(x) = \tfrac{1}{2}(x^2 - 4)^{-1/2}(2x) \Rightarrow f'(7) = \dfrac{7}{\sqrt{45}} = \dfrac{7}{3\sqrt{5}}$ and $g'(x) = g'(3) = 3$
15	$h(x) = \int_0^x f(t)dt \Rightarrow h'(x) = f(x)$ $h(6) = \int_0^6 f(t)dt < 0$ since below the $x-$axis; $h'(6) = f(6) = 0$; $h''(6) = f'(6) > 0$ since f increasing around 6. So, $h(6) < h'(6) < h''(6)$ $\boxed{A}$
16	$x(t) = (t-a)(t-b) = t^2 - at - bt + ab \Rightarrow x'(t) = v(t) = 2t - a - b = 0 \Rightarrow t = \dfrac{a+b}{2}$ $\boxed{B}$

17	$f(x)=\int_2^x g(t)\,dt \Rightarrow f'(x)=g(x)=$ a positive constant since f is increasing at a constant rate. $\boxed{A}$
18	$f(x)=\ln x \Rightarrow \lim_{h\to 0}\frac{\ln(4+h)-\ln(4)}{h}=f'(4)=\frac{1}{4}$ $\boxed{B}$
19	$f(x)=\frac{x}{x+2}\Rightarrow f'(x)=\frac{(x+2)(1)-x(1)}{(x+2)^2}=\frac{2}{(x+2)^2}=\frac{1}{2}\Rightarrow$ $(x+2)^2=4\Rightarrow x+2=\pm 2\Rightarrow x=0,-4\Rightarrow f(0)=0, f(-4)=2$ $\boxed{C}$
20	$f(x)=(2x+1)^3;\ f(0)=1$ so $g(1)=0;\ f'(x)=3(2x+1)^2 2=6(2x+1)^2\Rightarrow f'(0)=6$ $g'(x)=\frac{1}{f'[g(x)]}\Rightarrow g'(1)=\frac{1}{f'[g(1)]}=\frac{1}{f'[0]}=\frac{1}{6}$ $\boxed{D}$
21	horizontal asymptote: $\lim_{x\to\infty} y=5$ so take the limit of each: $\lim_{x\to\infty}\frac{20x^2-x}{1+4x^2}=5$ $\boxed{E}$
22	$f(x)=\frac{\ln x}{x}, x>0\Rightarrow f'(x)=\frac{x\left(\frac{1}{x}\right)-\ln x}{x^2}=\frac{1-\ln x}{x^2}=0\Rightarrow \ln x=1\Rightarrow x=e;\ f'>0, 0<x<e;\ f'<0, x>e$ So relative maximum and absolute maximum value is $f(e)=\frac{\ln e}{e}=\frac{1}{e}$ $\boxed{B}$
23	When $\frac{dP}{dt}$ is constant, then growth is constant and linear. $\boxed{A}$
24	$g(x)=x^2e^{kx}\Rightarrow g'(x)=2xe^{kx}k+e^{kx}2x\Rightarrow g'\left(\frac{2}{3}\right)=2\left(\frac{2}{3}\right)e^{(2/3)k}k+e^{(2/3)k}2\left(\frac{2}{3}\right)=0\Rightarrow$ $e^{(2/3)k}\left[\frac{4}{3}+\frac{4}{9}k\right]=0\Rightarrow \frac{4}{3}+\frac{4}{9}k=0\Rightarrow \frac{4}{9}k=-\frac{4}{3}\Rightarrow k=-3$ $\boxed{A}$
25	$\int dy=\int 2\sin x\,dx\Rightarrow y=-2\cos x+C\Rightarrow$ $y(\pi)=1,\ 1=-2\cos\pi+C\Rightarrow 1=-2(-1)+C\Rightarrow C=-1\Rightarrow y=-2\cos x-1$ $\boxed{E}$
26	Since e^{-t^2} is always positive, then $g'(2)=\int_0^2 e^{-t^2}dt>0$ so g is increasing on the interval. $g''(x)=e^{-x^2}>0$ so g is concave up on the interval. $\boxed{A}$
27	$\frac{dy}{dx}=\frac{2x-y}{x+2y}\Rightarrow \frac{d^2y}{dx^2}=\frac{(x+2y)(2-\frac{dy}{dx})-(2x-y)(1+2\frac{dy}{dx})}{(x+2y)^2}\Rightarrow \left.\frac{dy}{dx}\right\|_{(3,0)}=2$ $\left.\frac{d^2y}{dx^2}\right\|_{(3,0)}=\frac{(3)(0)-(6)(5)}{(3)^2}=-\frac{30}{9}=-\frac{10}{3}$ $\boxed{A}$
28	$v(t)=x'(t)=\cos t+\sin t=0\Rightarrow \tan t=-1\Rightarrow t=\frac{3\pi}{4};$ $a(t)=v'(t)=-\sin t+\cos t\Rightarrow a\left(\frac{3\pi}{4}\right)=-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}=-\sqrt{2}$ $\boxed{A}$

76 $f'+ \Rightarrow f$ increasing $\Rightarrow c,e$; f'increasing $\Rightarrow f''$ concave up $\Rightarrow d,e$ [E]

77 From the IVT, $f(x)=13$ for some x on $(2,4)$ since $f(2)=10$ and $f(4)=20$. [A]

78 $y=e^{\tan x}-2=0 \Rightarrow e^{\tan x}=2 \Rightarrow \tan x=\ln 2 \Rightarrow x=\tan^{-1}(\ln 2)=0.60611193 \Rightarrow$ store as "a"

Using calculator: $y'(\text{a})=2.9609 \approx 2.961$ [D]

79 Average value of the velocity on $[0,8]=\dfrac{1}{8-0}\int_0^8 v(t)\,dt$ [B]

80 I. f' changes from $-$ to $+$ at $x=-3$: true II. f' changes from inc to dec or dec to inc at $x=-2$: false

III. f' decreases on $0<x<4$: true [E]

81 Water in the tank = initial amount + amount pumped in $= 800+\int_0^{20} r(t)\,dt \approx 1220\,gal$ [D]

82 Graph $f'(x)$ and find that the relative maximum occurs at $x=0.350$ [C]

83 An estimate for the distance traveled is an estimate of the area below the curve on the interval $0 \le x \le 8$.

One estimate would be to connect v(0) and v(8) with a straight line and use the

area of the triangle as the estimate: $\frac{1}{2}(8)(50)=200$ close to 210 [D]

84 $f''(x)=(4x^3-4x)e^{(x^4-2x^2+1)}$. Graph $f''(x)$ and find where it is below the $x-axis$: $(-1.5,-1)$ and $(0,1)$ [D]

85 f has a relative maximum at $x=1$ because f' changes from + to - there. [C]

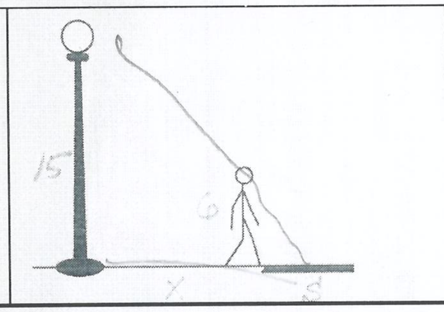
86 $f'>0$ so f increases for all x and $\int_4^7 f(t)\,dt=0$ so f is both negative and positive on $[4,7] \Rightarrow$ so not A, D, or E

For C: slope $\dfrac{3-6}{7-5}=-\dfrac{3}{2}<0$ So, [B]

87 f is concave down when $f''<0$ or when its graph is below the $x-axis$: $-2<x<-1$ and $1<x<3$ [E]

88 $\dfrac{15}{x+s}=\dfrac{6}{s} \Rightarrow 15s=6x+6s \Rightarrow 9s=6x \Rightarrow s=\dfrac{2}{3}x \Rightarrow$

$\dfrac{ds}{dt}=\dfrac{2}{3}\dfrac{dx}{dt}=\dfrac{2}{3}(4)=\dfrac{8}{3}\dfrac{ft}{sec}=2.667\dfrac{ft}{sec}$ [B]



89 $v(3)=v(0)+\int_0^3 v'(t)\,dt=v(0)+\int_0^3 a(t)\,dt=5+6.710=11.710$ [E]

90 Let $t=2x$, so $dt=2dx \Rightarrow dx=\frac{1}{2}dt$ and $x=12 \Rightarrow t=2(12)=24$; $x=6 \Rightarrow t=2(6)=12$

$\int_6^{12} f(2x)\,dx=10 \Rightarrow \frac{1}{2}\int_{12}^{24} f(t)\,dt=10 \Rightarrow \int_{12}^{24} f(t)\,dt=20$ [B]

91 f' changed from decreasing to increasing somewhere on $-2<x<3$, and increasing to decreasing somewhere

on $3<x<6$. So f'' changed signs at least once on these intervals $\Rightarrow f$ has at least 2 inflection points [B]

92 Area of the cross sectional square $= s^2=\left(\sqrt{x}-x^2\right)^2$

Volume $=\int_0^1\left(\sqrt{x}-x^2\right)^2 dx=0.12857 \approx 0.129$ [A]