2013 AB
Calculator

1 $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$ $0 \le t \le 8$ $t=0,\ 500$

40/BC rate gravel unprocessed gravel arrives

a) $G'(5) = \boxed{-24.588}$ tons/h^2 1

The rate the gravel arriving changes at a rate of -24.588 tons/hr^2 (or decreases at 24.588 tons/h^2) 1

b) $\int_0^8 90 + 45\cos\left(\frac{t^2}{18}\right)dt$ 1 (total that arrives)

$= \boxed{825.551 \text{ tons}}$ 1

*c) $G(5) = 98.141$ tons/hr (unprocessed) 1 100 tons/hr (processed)

$98.141 - 100 < 0$ therefore unprocessed gravel is $\boxed{\text{decreasing}}$ 1

$G(t) - 100 = 0$ rate gravel enters − rate proc.

*d) $G(t) = 100$ at 4.923 (rate gravel enters − rate processed = 0)

max amount of unprocessed gravel occurs at endpoints $t=0$, $t=8$ or at $t=4.923$.

1 t=0 500

1 $t = 4.923$ $500 + \int_0^{4.923} [G(t) - 100]\,dt = 635.376$ tons 1

$t=8$ $500 + \int_0^8 (G(t) - 100)\,dt = 525.551$ tons

max amount of unprocessed gravel is

$\boxed{635.376 \text{ tons}}$

Global 3.3

$$10 + \int_0^t v(t)\,dt$$

(2) $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$ $\quad s(0) = 10$

a) speed $= |v(t)| = 2$ (1) graph and find intersection

$\boxed{t = 3.128,\ 3.473}$ (1)

b) (1) $10 + \int_0^5 (-2 + (t^2+3t)^{6/5} - t^3)\,dt = \boxed{-9.207}$ (1)

c) $-2 + (t^2 + 3t)^{6/5} - t^3 = 0$ (1)

$\boxed{t = .536,\ 3.318}$ (1) particle changes (1) direction where velocity changes from positive to negative or negative to positive

* d) $v(4) = -11.476$ (1)

$v'(4) = a(4) = -22.296$

Since the velocity and acceleration are both negative, the speed of the particle is (1) $\boxed{\text{Increasing}}$

$\boxed{2.55}$

~~No~~ Calculator

AB/BC

③ a) $C'(3.5) \approx \frac{12.8-11.2}{4-3} = \boxed{1.6 \text{ ounces per minute}}$ ①①

b) Yes. $C'(t) = \frac{C(4)-C(2)}{4-2} = \frac{12.8-8.8}{2} = 2$ ① MVT ① applies - continuous, differentiable

There is atleast 1 place where $C'(t)=2$

c) $\frac{1}{6}\left(2(5.3) + 2(11.2) + 2(13.8)\right) =$

$\frac{1}{6}(10.6 + 22.4 + 27.6) = \frac{1}{6}(60.6) = \boxed{10.1 \text{ ounces}}$ ①①

$\frac{1}{6}\int_0^6 C(t)\,dt$ is the average number of ounces ①

of coffee that has dripped through the coffee filter at the end of 6 minutes.

d) $B(t) = 16 - 16e^{-.4t}$

$B'(t) = -16e^{-.4t}(-.4) = 6.4e^{-.4t}$ ①

$B'(5) = \boxed{6.4e^{-2} \text{ ounces/min}}$ ①

$\frac{6.4}{e^2}$

$\boxed{3.65}$

no calc

average 2.62

AB/BC

4) a) f has a min where f' changes from negative to positive.
This occurs at $\boxed{x=6}$ 1

* b) abs. min occurs at 6 or at endpoints,
x=0, or x=8 1

$f(8) - \quad f(0)$

$f(0) = f(8) - \int_0^8 f'(x) = 4 - (2+6-3+7) = -8$ 1
$f(6) = f(8) - \int_6^8 f'(x) = 4 - (7) = -3$
$f(8) = 4$
Absolute min value is $\boxed{-8}$ 1

c) ccd → f'' negative → f' decreasing
Inc → f' positive 1
$\boxed{(0,1)\ (3,4)}$ 1

need connection to 1st deriv.

d) $g(x) = (f(x))^3$
$g'(x) = 3[f(x)]^2 f'(x)$ 1 2

$g'(3) = 3[f(3)]^2 f'(3)$
$= 3(-5/2)^2 (4) = 3 \cdot \frac{25}{4} \cdot 4 = \boxed{75}$ 1

2.62

no calc | average 4.14

5) $f(x) = 2x^2 - 6x + 4$
$g(x) = 4\cos(\frac{1}{4}\pi x)$

a) $A = \int_0^2 [4\cos(\frac{1}{4}\pi x) - (2x^2 - 6x + 4)]\, dx$ 1

$u = \frac{1}{4}\pi x$
$du = \frac{1}{4}\pi = \pi/4$

$\frac{4}{\pi} \cdot 4 \int_0^2 \cos u\, du$

$\frac{16}{\pi} \int_0^{2} \cos u\, du - \int_0^2 2x^2 - 6x + 4$

1 $\frac{16}{\pi} \sin(\frac{1}{4}\pi x) \Big|_0^2 - (\frac{2x^3}{3} - 3x^2 + 4x) \Big|_0^2$ 1

$\frac{16}{\pi}[\sin(\frac{\pi}{2}) - \sin(0)] - [(\frac{16}{3} - 12 + 8) - 0]$

$\frac{16}{3} + \frac{36}{3} - \frac{24}{3} = -4/3$

$\boxed{\frac{16}{\pi} - \frac{4}{3}}$ 1 $\quad \frac{48 - 4\pi}{3\pi}$

b) 1 $\pi \int_0^2 [(4 - (2x^2 - 6x + 4))^2 - (4 - 4\cos(\frac{1}{4}\pi x))^2]\, dx$ 1 1

c) $\int_0^2 [4\cos(\frac{1}{4}\pi x) - (2x^2 - 6x + 4)]^2\, dx$ 1 1

$\boxed{4.14}$

no calc

$\boxed{\text{average } 3.17}$

6) $\frac{dy}{dx} = e^y(3x^2-6x)$

a) $(1,0)$ $\frac{dy}{dx} = \boxed{-3}$ 1

$y - 0 = -3(x-1)$

$\boxed{y = -3(x-1)}$ 1

$y = e^{y}(3x^2-6x)(x-1)$

$y = -3(1.2-1)$

$y = -3(.2) = \boxed{-.6}$ 1

b) $\frac{dy}{dx} = e^y(3x^2-6x)$

$\int \frac{1}{e^y} dy = \int 3x^2 - 6x \; dx$ 1

$u = -y$
$du = -1$

$\int e^{-y} = x^3 - 3x^2 + C$ 1 1

$-\int e^u$

1 $-e^{-y} = x^3 - 3x^2 + C$

$-1 = -2 + C \qquad C = 1$ 1

$-e^{-y} = x^3 - 3x^2 + 1$

$e^{-y} = -x^3 + 3x^2 - 1$

$-y = \ln(-x^3 + 3x^2 - 1)$

$\boxed{y = -\ln(-x^3+3x^2-1)}$ 1 or $y = \ln \frac{1}{-x^3+3x^2-1}$

$y = \ln \frac{-1}{x^3 - 3x^2 + 1}$

$\boxed{3.17}$