Calculus AB practice exam - no calculator
2013

1. $\lim_{x\to 2}\dfrac{x^2+x-6}{x^2-4}$ is

$\dfrac{(x+3)(x-2)}{(x+2)(x-2)} = 5/4$

(A) $-\frac{1}{4}$ (B) 0 (C) 1 (D) $\frac{5}{4}$ (E) nonexistent

2. If $f(x) = x^3 - x^2 + x - 1$, then $f'(2) =$

$3x^2 - 2x + 1$

$12 - 4 + 1 = 9$

(A) 10 (B) 9 (C) 7 (D) 5 (E) 3

3. Which of the following definite integrals has the same value as $\int_0^4 xe^{x^2}\,dx$?

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}\int e^u\,du \rightarrow \frac{1}{2}e^u\Big|_0^4$

$x=0 \rightarrow u=0$

$x=4 \rightarrow u=16$

$\frac{1}{2}\int_0^{16} e^u\,du$

(A) $\frac{1}{2}\int_0^4 e^u\,du$

(B) $\frac{1}{2}\int_0^{16} e^u\,du$

(C) $2\int_0^2 e^u\,du$

(D) $2\int_0^4 e^u\,du$

(E) $2\int_0^{16} e^u\,du$

4. Which of the following is an equation of the line tangent to the graph of $x^2 - 3xy = 10$ at the point $(1, -3)$?

$2x - 3x\frac{dy}{dx} + y(-3) = 0$

$\frac{dy}{dx} = \frac{3y - 2x}{-3x} = \frac{-9-2}{-3} = \frac{-11}{-3} = 11/3$

$y + 3 = 11/3(x-1)$

(A) $y + 3 = -11(x - 1)$

(B) $y + 3 = -\frac{7}{3}(x - 1)$

(C) $y + 3 = \frac{1}{3}(x - 1)$

(D) $y + 3 = \frac{7}{3}(x - 1)$

(E) $y + 3 = \frac{11}{3}(x - 1)$

5. If g is the function given by $g(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 70x + 5$, on which of the following intervals is g decreasing?

$g'(x) = x^2 + 3x - 70$

$(x+10)(x-7)$

+ | − | +

−10 7

$g'(x)$ neg $\longleftrightarrow$ $g(x)$ dec

(A) $(-\infty, -10)$ and $(7, \infty)$

(B) $(-\infty, -7)$ and $(10, \infty)$

(C) $(-\infty, 10)$

(D) $(-10, 7)$

(E) $(-7, 10)$

6. $\int_2^4 \dfrac{dx}{5-3x} =$

$u = 5 - 3x$

$du = -3$

$-\frac{1}{3}\ln|5-3x|\Big|_2^4 = -\frac{1}{3}(\ln 7 - 0)$

(A) $-\ln 7$ (B) $-\frac{\ln 7}{3}$ (C) $\frac{\ln 7}{3}$ (D) $\ln 7$ (E) $3\ln 7$

7. Let f be the function given by $f(x) = x^3 - 6x^2 + 8x - 2$. What is the instantaneous rate of change of f at $x = 3$?

$f'(x) = 3x^2 - 12x + 8$

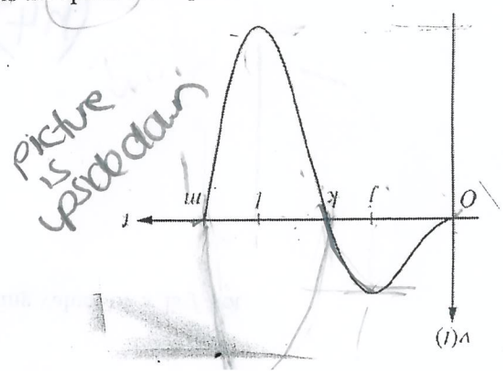
$f'(3) = 27 - 36 + 8 = -1$

(A) −5 (B) $-\frac{15}{4}$ (C) −1 (D) 6 (E) 17

8. A particle moves along a straight line. The graph of the particle's velocity $v(t)$ at time t is shown above for $0 \le t \le m$, where j, k, l, and m are constants. The graph intersects the horizontal axis at $t = 0$, $t = k$, and $t = m$ and has horizontal tangents at $t = j$ and $t = l$. For what values of t is the speed of the particle decreasing?

speed = $|v(t)|$ dec

picture is upside down



(A) $j \le t \le l$

(B) $k \le t \le m$

(C) $j \le t \le k$ and $l \le t \le m$

(D) $0 \le t \le j$ and $k \le t \le l$

(E) $0 \le t \le j$ and $l \le t \le m$

9. Let f be the function given by $f(x) = \dfrac{(x-2)^2(x+3)}{(x-2)(x+1)}$. For which of the following values of x is f not continuous?

(A) -3 and -1 only

(B) -3, -1, and 2

(C) -1 only

(D) -1 and 2 only

(E) 2 only

hole $x=2$
VA $x=-1$

(+,+)

10. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 - 4$ for time $t \geq 0$. If the particle is at position $x = -2$ at time $t = 0$, what is the position of the particle at time $t = 3$?

(A) 13 (B) 15 (C) 16 (D) 17 (E) 25

$\int 3t^2 - 4$

$x(t) = t^3 - 4t + C = -2$

$x(t) = t^3 - 4t - 2$

$x(3) = 27 - 12 - 2$

11. Let f be the function defined by $f(x) = \int_0^x (2t^3 - 15t^2 + 36t)\,dt$. On which of the following intervals is the graph of $y = f(x)$ concave down?

(A) $(-\infty, 0)$ only

(B) $(-\infty, 2)$

(C) $(0, \infty)$

(D) $(2, 3)$ only

(E) $(3, \infty)$ only

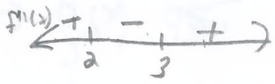
$f'(x)$

$f''(x) = 6t^2 - 30t + 36 = 0$

$6(t^2 - 5t + 6) = 0$

$(t-3)(t-2) = 0$

$t = 3 \quad t = 2$



12. For which of the following does $\lim_{x \to \infty} f(x) = 0$?

I. $f(x) = \dfrac{\ln x}{x^{99}}$ ✓

II. $f(x) = \dfrac{e^x}{\ln x}$ ✗

III. ✓ $f(x) = \dfrac{x^{99}}{e^x}$

(A) I only

(B) II only

(C) III only

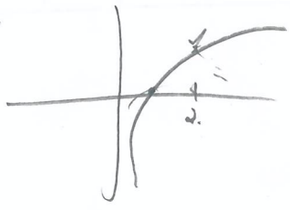
(D) I and II only

(E) I and III only

e^x $\ln x$

x^{99}

exponential always increases faster than polynomial



(0, −5) (2, −10)

slopes < 3

$\dfrac{-10+5}{2}$

$\dfrac{-5}{2}$

-2.5

13. Let f be a differentiable function such that $f(0) = -5$ and $f'(x) \leq 3$ for all x. Of the following, which is not a possible value for $f(2)$?

(A) -10 (B) -5 (C) 0 (D) 1 (E) 2

14. Let f be the function given above. What are all values of a and b for which f is differentiable at $x = 1$?

(A) $a = \dfrac{1}{2}$ and $b = -\dfrac{1}{2}$

(B) $a = \dfrac{1}{2}$ and $b = \dfrac{3}{2}$

(C) $a = \dfrac{1}{2}$ and b is any real number

(D) $a = b + 1$, where b is any real number

(E) There are no such values of a and b.

(2, −10)

deriv

$f(x) = \begin{cases} x + b & \text{if } x \leq 1 \\ ax^2 & \text{if } x > 1 \end{cases}$

$1 = 2a(1)$

$1 = 2a$

$\frac{1}{2} = a$

cont.

$1 + b = a$

$1 + b = \frac{1}{2}$

$b = -\frac{1}{2}$

15. The table above gives values for the functions f and g and their derivatives at $x = 3$. Let k be the function given by $k(x) = \dfrac{f(x)}{g(x)}$, where $g(x) \neq 0$. What is the value of $k'(3)$?

$f(3)$	$g(3)$	$f'(3)$	$g'(3)$
−1	2	5	−2

(A) $-\dfrac{5}{2}$ (B) -2 (C) 2 (D) 3 (E) 8

$k'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$\dfrac{2(5) - (-1)(-2)}{4} = \dfrac{10-2}{4} = 2$

16. If $y = 5x\sqrt{x^2+1}$, then $\dfrac{dy}{dx}$ at $x = 3$ is

(A) $\dfrac{5}{2\sqrt{10}}$ (B) $\dfrac{15}{\sqrt{10}}$ (C) $\dfrac{15}{2\sqrt{10}} + 5\sqrt{10}$ (D) $\dfrac{45}{\sqrt{10}} + 5\sqrt{10}$ (E) $\dfrac{45}{\sqrt{10}} + 15\sqrt{10}$

$\dfrac{dy}{dx} = 5x \cdot \dfrac{1}{2}(x^2+1)^{-1/2}(2x) + \sqrt{x^2+1}(5)$

$= 45/\sqrt{10} + 5\sqrt{10}$

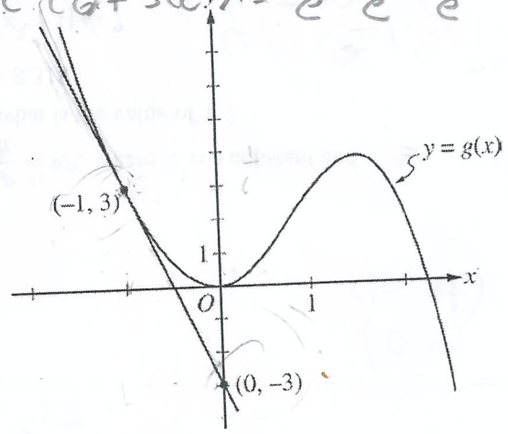
17. If $\lim_{h\to 0}\dfrac{\arcsin(a+h)-\arcsin(a)}{h}=2$, which of the following could be the value of a?

(A) $\dfrac{\sqrt{2}}{2}$ (B) $\dfrac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\dfrac{1}{2}$ (E) 2

18. If $\ln(2x+y)=x+1$, then $\dfrac{dy}{dx}=$

(A) -2 (B) $2x+y-2$ (C) $2x+y$ (D) $4x+2y-2$ (E) $y-\dfrac{y}{x}$

19. The figure above shows the graph of the function g and the line tangent to the graph of g at $x=-1$. Let h be the function given by $h(x)=e^x\cdot g(x)$. What is the value of $h'(-1)$?



(A) $\dfrac{9}{e}$ (B) $\dfrac{-3}{e}$ (C) $\dfrac{-6}{e}$ (D) $\dfrac{-6}{e}-\dfrac{3}{e^2}$ (E) -6

20. For $x>0$, $\dfrac{d}{dx}\left(\int_0^{2x}\ln\left(t^3+1\right)dt\right)=$

(A) $\ln\left(x^3+1\right)$

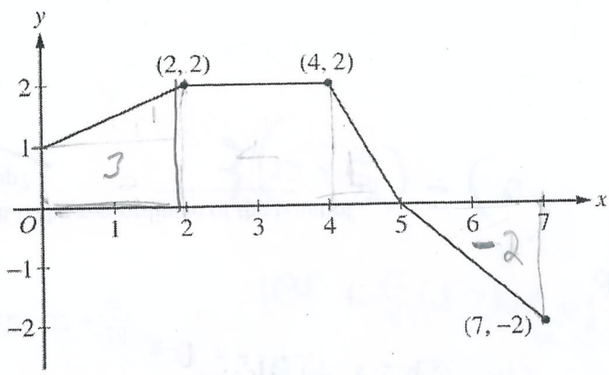
(B) $\ln\left(8x^3+1\right)$

(C) $2\ln\left(x^3+1\right)$

(D) $2\ln\left(8x^3+1\right)$

(E) $24x^2\ln\left(8x^3+1\right)$

21. The graph of a function f is shown above. What is the value of $\int_0^7 f(x)\,dx$?



Graph of f

(A) 6 (B) 8 (C) 10 (D) 14 (E) 18

22. The function f is continuous for all real numbers, and the average rate of change of f on the closed interval $[6, 9]$ is $-\dfrac{3}{2}$. For $6<c<9$, there is no value of c such that $f'(c)=-\dfrac{3}{2}$. Of the following, which must be true?

(A) $\dfrac{1}{3}\int_6^9 f(x)\,dx=-\dfrac{3}{2}$

(B) $\int_6^9 f(x)\,dx$ does not exist.

(C) $\dfrac{f'(6)+f'(9)}{2}=-\dfrac{3}{2}$

(D) $f'(x)<0$ for all x in the open interval $(6, 9)$.

(E) f is not differentiable on the open interval $(6, 9)$.

23. Let f be the function defined by $f(x)=2x+e^x$. If $g(x)=f^{-1}(x)$ for all x and the point $(0, 1)$ is on the graph of f, what is the value of $g'(1)$?

(A) $\dfrac{1}{2+e}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ (D) 3 (E) $2+e$

24. The function g is given by $g(x)=4x^3+3x^2-6x+1$. What is the absolute minimum value of g on the closed interval $[-2, 1]$?

(A) -7 (B) $-\dfrac{3}{4}$ (C) 0 (D) 2 (E) 6

25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

(C) $y = -\ln\left(-e^x + 5\right)$

(D) $y = -\ln\left(e^x + 3\right)$

(E) $y = \ln\left(e^x + 3\right)$

26. Which of the following is an antiderivative of $f(x) = \sqrt{1 + x^3}$?

(A) $\frac{2}{3}\left(1 + x^3\right)^{3/2}$

(B) $\frac{\frac{2}{3}\left(1 + x^3\right)^{3/2}}{3x^2}$

(C) $\int_0^{1+x^3} \sqrt{t}\, dt$

(D) $\int_0^{x^3} \sqrt{1 + t}\, dt$

(E) $\int_0^{x} \sqrt{1 + t^3}\, dt$

27. For time $t \geq 0$, the height h of an object suspended from a spring is given by $h(t) = 16 + 7\cos\left(\frac{\pi t}{4}\right)$. What is the average height of the object from $t = 0$ to $t = 2$?

(A) 16 (B) $\frac{39}{2}$ (C) $16 - \frac{14}{\pi}$ (D) $16 + \frac{14}{\pi}$ (E) $32 + \frac{28}{\pi}$

28. The function f is defined by $f(x) = \sin x + \cos x$ for $0 \leq x \leq 2\pi$. What is the x-coordinate of the point of inflection where the graph of f changes from concave down to concave up?

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$ (E) $\frac{9\pi}{4}$

90. The population P of a city grows according to the differential equation $\frac{dP}{dt} = kP$, where k is a constant and t is measured in years. If the population of the city doubles every 12 years, what is the value of k ?

(A) 0.058 (B) 0.061 (C) 0.167 (D) 0.693 (E) 8.318

91. The function f is continuous and $\int_0^8 f(u)\, du = 6$. What is the value of $\int_1^3 xf\left(x^2 - 1\right) dx$?

(A) $\frac{3}{2}$ (B) 3 (C) 6 (D) 12 (E) 24

92. The function f is defined for all x in the closed interval $[a, b]$. If f does not attain a maximum value on $[a, b]$, which of the following must be true?

(A) f is not continuous on $[a, b]$.

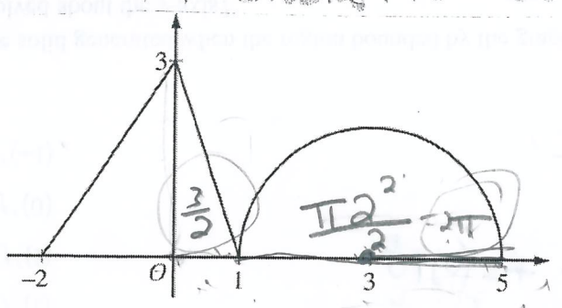
(B) f is not bounded on $[a, b]$.

(C) f does not attain a minimum value on $[a, b]$.

(D) The graph of f has a vertical asymptote in the interval $[a, b]$.

(E) The equation $f'(x) = 0$ does not have a solution in the interval $[a, b]$.

2013 College Board AP Calculus AB Practice Exam Calculator



Graph of f

76. The graph of the function f shown above consists of two line segments and a semicircle. Let g be defined by $g(x) = \int_0^x f(t)\,dt$. What is the value of $g(5)$?

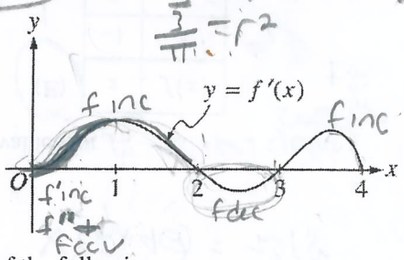
(A) 0 (B) $-1.5 + 2\pi$ (C) 2π (D) $1.5 + 2\pi$ (E) $4.5 + 2\pi$

77. The volume of a sphere is decreasing at a constant rate of 3 cubic centimeters per second. At the instant when the radius of the sphere is decreasing at a rate of 0.25 centimeter per second, what is the radius of the sphere? (The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(A) 0.141 cm (B) 0.244 cm (C) 0.250 cm (D) 0.489 cm (E) 0.977 cm

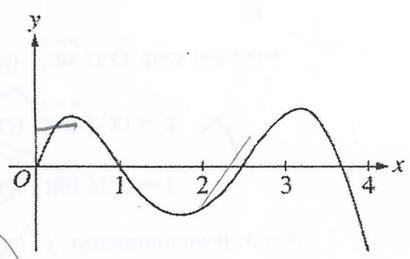
78. Let f and g be continuous functions such that $\int_0^{10} f(x)\,dx = 21$, $\int_0^{10} \frac{1}{2}g(x)\,dx = 8$, and $\int_3^{10} (f(x) - g(x))\,dx = 2$. What is the value of $\int_0^3 (f(x) - g(x))\,dx$?

(A) 3 (B) 7 (C) 11 (D) 15 (E) 19



79. The figure above shows the graph of f', the derivative of the function f. If $f(0) = 0$, which of the following could be the graph of f?

(A)



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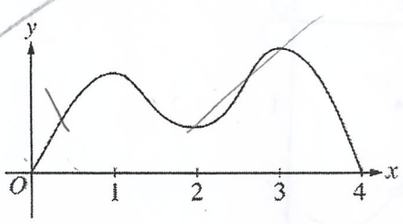
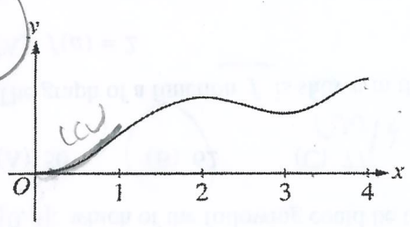
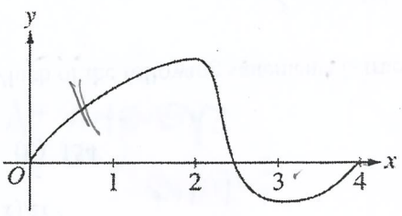


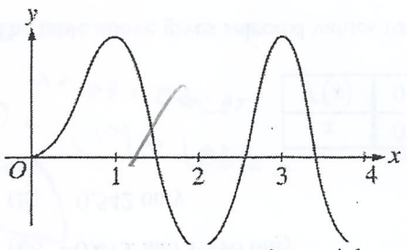
(C)



(D)



(E)



80. For time $t \geq 0$, the position of a particle traveling along a line is given by a differentiable function s. If s is increasing for $0 \leq t < 2$ and s is decreasing for $t > 2$, which of the following is the total distance the particle travels for $0 \leq t \leq 5$?

(A) $s(0) + \int_0^2 s'(t)\,dt - \int_2^5 s'(t)\,dt$

(B) $s(0) + \int_2^5 s'(t)\,dt - \int_0^2 s'(t)\,dt$

(C) $\int_2^5 s'(t)\,dt - \int_0^2 s'(t)\,dt$

(D) $\left|\int_0^5 s'(t)\,dt\right|$

(E) $\int_0^5 |s'(t)|\,dt$

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81. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit (°F). If the initial temperature of the tea, at time $t = 0$ minutes, is 200°F and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

(A) 175°F (B) 130°F (C) 95°F (D) 70°F (E) 45°F

82. The derivative of the function f is given by $f'(x) = x^3 - 4\sin(x^2) + 1$. On the interval $(-2.5, 2.5)$, at which of the following values of x does f have a relative maximum?

(A) −1.970 and 0

(B) −1.467 and 1.075

(C) −0.475, 0.542, and 1.396

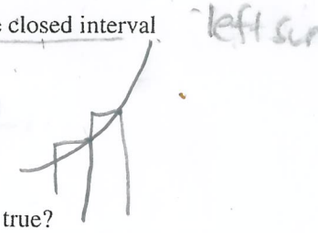
(D) −0.475 and 1.396 only

(E) 0.542 only

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0	4	10	18	28	40	54

83. The table above gives selected values for a continuous function f. If f is increasing over the closed interval $[0, 3]$, which of the following could be the value of $\int_0^3 f(x)\,dx$?

(A) 50 (B) 62 (C) 77 (D) 100 (E) 154



84. The graph of a function f is shown in the figure above. Which of the following statements is true?

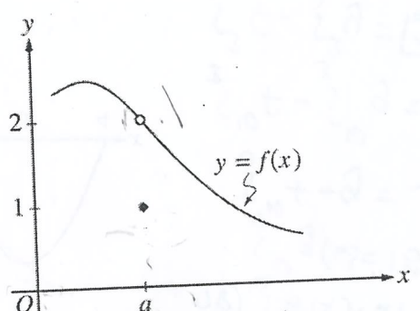
(A) $f(a) = 2$

(B) f is continuous at $x = a$.

(C) $\lim_{x \to a} f(x) = 1$

(D) $\lim_{x \to a} f(x) = 2$

(E) $\lim_{x \to a} f(x)$ does not exist.



85. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = \cos\sqrt{t}$. What is the velocity of the particle at the first instance the particle is at the origin?

(A) −1 (B) −0.624 (C) −0.318 (D) 0 (E) 0.065

86. If $f'(x) > 0$ for all x and $f''(x) < 0$ for all x, which of the following could be a table of values for f?

(A)

x	$f(x)$
−1	4
0	3
1	1

(B)

x	$f(x)$
−1	4
0	4
1	4

(C)

x	$f(x)$
−1	4
0	5
1	6

(D)

x	$f(x)$
−1	4
0	5
1	7

(E)

x	$f(x)$
−1	4
0	6
1	7

87. Let f be the function with first derivative given by $f'(x) = (3 - 2x - x^2)\sin(2x - 3)$. How many relative extrema does f have on the open interval $-4 < x < 2$?

(A) Two (B) Three (C) Four (D) Five (E) Six

88. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

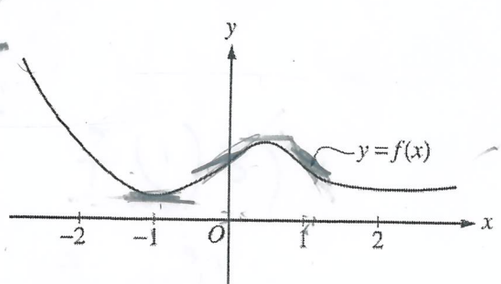
(A) $f'(-1) < f'(1) < f'(0)$

(B) $f'(-1) < f'(0) < f'(1)$

(C) $f'(0) < f'(-1) < f'(1)$

(D) $f'(1) < f'(-1) < f'(0)$

(E) $f'(1) < f'(0) < f'(-1)$



89. What is the volume of the solid generated when the region bounded by the graph of $x = \sqrt{y - 2}$ and the lines $x = 0$ and $y = 5$ is revolved about the y-axis?

(A) 3.464 (B) 4.500 (C) 7.854 (D) 10.883 (E) 14.137