2010 AP Calculus AB - calculator

AB|BC 1) $f(t) = 7t\,e^{\cos t}$ (rate snow accumulates from 12 - 9AM)

$g(t) = \begin{cases} 0 & 0 \le t < 6 \\ 125 & 6 \le t < 7 \\ 108 & 7 \le t \le 9 \end{cases}$ rates snow removed

a) $\int_0^6 f(t)\,dt = \int_0^6 7t\,e^{\cos t}\,dt = \boxed{142.275}$ 1 1

b) rate of accumulated snow − rate of removing snow (at 8AM)

$f(8) - g(8)$ $7te^{\cos t} - 108 =$

$48.417 - 108 = \boxed{-59.583 \text{ ft per hour}}$ 1

c) $h(t) = \begin{cases} 0 & 0 \le t < 6 \\ \int_6 125 & 6 \le t < 7 \\ \int_6^9 108 & 7 \le t \le 9 \end{cases} \Rightarrow \begin{cases} 0 & 0 \le t < 6 \\ 125(t-6) & 6 \le t < 7 \\ 125 + 108(t-7) & 7 \le t \le 9 \end{cases}$ 1 1 1

d) $\underbrace{\int_0^9 7te^{\cos t}\,dt}_{\text{amount accumulated}} - \underbrace{\left[\int_6^7 125\,dt + \int_7^9 108\,dt\right]}_{\text{amount removed}}$ 1 1

$367.335 - (125 + 216) = \boxed{26.335 \text{ cubic ft}}$ 1

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AB/BC 2) a) $\frac{E(7)-E(5)}{7-5} = \frac{21-13}{7-5} = \frac{8}{2} = \boxed{4}$ 1 400 entries per hour

b) $\frac{1}{8}\left[\frac{1}{2}(2)(0+4) + \frac{1}{2}(3)(4+13) + \frac{1}{2}(2)(13+21) + \frac{1}{2}(1)(21+23)\right]$ 1

$\frac{1}{8}\left[4 + \frac{51}{2} + 34 + 22\right] = \frac{1}{8}(85.8) = \boxed{10.688}$ 1

$\frac{1}{8}\int_0^8 E(t)$ is the average number of entries (in hundreds) per hour from noon until 8pm. 1

c) 1 $\int_8^{12} t^3 - 30t^2 + 298t - 976$ (# processed from 8 PM - midnight)

= 16 (hundred)

2300 (total entries) − 1600 (processed) = $\boxed{700}$ 1 left to process

d) processed most quickly when $P(t)$ is at its max

($P(t)$ changes from increasing to decreasing)

This will occur at $t=8$ or $t=12$ or a critical point of P.

$P'(t) = 3t^2 - 60t + 298 = 0$ 1 $\rightarrow t = 9.184, 10.817$

$P(8) = 0$ $P(9.184) = 5.087$ $P(10.817) = 2.91$ 1

$P(12) = 8$

1 $\boxed{t=12}$ or midnight is when entries are processed most quickly.

3 - calculator

(rate snow accumulates from 12 - 9AM)

$$\left.\begin{array}{l} <6 \\ <7 \\ \le 9 \end{array}\right\} \text{rates snow removed}$$

$$\int 7te^{\cos t}\,dt = \boxed{142.275}$$ (1) (1)

...ted snow − rate of removing snow (at 8 PM)

108 =

$= \boxed{-59.583 \text{ ft per hour}}$ (1)

$$\begin{array}{ll} \le t < 6 \\ \le t < 7 \\ 7 \le t \le 9 \end{array} \Rightarrow \begin{cases} 0 & 0 \le t < 6 \\ 125(t-6) & 6 \le t < 7 \\ 125 + 108(t-7) & 7 \le t \le 9 \end{cases}$$ (1) (1) (1)

$$- \underbrace{\left[\int_6^7 125\,dt + \int_7^9 108\,dt\right]}_{\text{amount removed}}$$ (1)

$- (125 + 216) = \boxed{26.335 \text{ cubic ft}}$ (1)

per hour

1+23] (1)

] (1)

hundreds) per

8 PM - midnight)

0 left

process

ax

)

10.817

) = 291 (1)

e

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3) a) $\int_0^3 r(t)\,dt = \frac{1}{2}(2)(1000+1200) + \frac{1}{2}(1)(1200+800)$ (1)

$= 2200 + 1000 = \boxed{3200 \text{ people}}$ (1)

b) Since $r(t)$ is above 800 between 2 and 3 hrs, the number of people in line is $\boxed{\text{increasing}}$ (1) (must have reason)

c) Line is longest when # of people in line reaches its max. This happens when $r(t)$ changes from from + to − (compared to 800 per hr). This occurs at $\boxed{t = 3}$. (1)

(1) $700 + \int_0^3 r(t) - 800(3)$ (1)

$700 + 3200 - 2400 = \boxed{1500 \text{ people}}$

d) $700 + \underbrace{\int_0^t r(t)}_{(1)} - 800t = 0$ (1) (1)

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4) a) Area R $= \int_0^9 (6 - 2\sqrt{x})\,dx$ 1 $= \int_0^9 6 - 2x^{1/2}$

$= 6x - \frac{4}{3}x^{3/2} \Big|_0^9 = (54 - 36) - (0 - 0) = \boxed{18}$ 1 1

b) $\boxed{\text{Volume } R = \pi \int_0^9 [\underbrace{(7-2\sqrt{x})^2 - (7-6)^2}_{2}]\,dx}$ 1

c) $V = \int_0^6 \underbrace{\left(\frac{y^2}{4} - 0\right)}_{\text{base}} \underbrace{\left(3\left(\frac{y^2}{4} - 0\right)\right)}_{\text{height}} dy$ 2 $y = 2\sqrt{x}$ $y/2 = \sqrt{x}$ $x = \frac{y^2}{4}$

$= \boxed{3\int_0^6 \left(\frac{y^2}{4}\right)^2 dy}$ 1 $3\int_0^6 \frac{y^4}{16}\,dy$ $\frac{3}{16}\int_0^6 y^4\,dy$

5) a) $g(3) = g(0) + \int_0^3 g'(x) = 5 + \frac{1}{4}\pi(2)^2 + \frac{1}{2}(1)(3)$ 1

$5 + \pi + \frac{3}{2} = \boxed{\frac{13}{2} + \pi}$ 1

$g(-2) = g(0) - \int_{-2}^0 g'(x) = \boxed{5 - \pi}$ 1

b) $y = g(x)$ has a point of inflection at 1 $x = 0$, 2 and 3. This is where $g'(x)$ changes from increasing to decreasing or decreasing to increasing 1

* c) $h(x) = g(x) - \frac{1}{2}x^2$

$h'(x) = g'(x) - x$ 1 $g'(x) = x$

At $x = 3$ $h'(x) = 3(x-2) - x = 2x - 6$ neg $[2,3)$ $g'(x) = 3(x-2)$

$h'(x) = -2x + 9 - x = -3x + 9$ neg $(3,5]$ $g'(x) = -1 - 2(x-5)$

1

1 neither a max or min at $x = 3$ since $g'(x)$ doesn't change signs.

for $[-2,2]$ $x^2 + y^2 = 4$ $y = g'(x) = \sqrt{4 - x^2} = x$ $4 - x^2 = x^2$ $x = \sqrt{2}$

$h''(x) = g''(x) - 1$ $h''(\sqrt{2}) = g''(\sqrt{2}) - 1 < 0$ since $g''(\sqrt{2}) < 0$

because $g'(x)$ is decreasing from $x = 0$ to $x = 2$

h has a max at $x = \sqrt{2}$ because $g'(\sqrt{2}) = 0$ $g''(\sqrt{2}) < 0$ 1

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6) a) $\frac{dy}{dx} = xy^3$ $\quad f(1) = 2$ $\quad m = 8$ 1

$\boxed{y - 2 = 8(x-1)}$ 1

b) $f(1.1)$ $\quad y - 2 = 8(1.1 - 1)$ 1

$y - 2 = 8.8 - 8 \rightarrow y - 2 = .8 \rightarrow \boxed{y = 2.8}$

$\frac{d^2y}{dy^2} = 104 > 0$ f(x) is concave up so tangent line lies below. 1

Approx is $\boxed{\text{less then}}$ actual value

c) $\frac{dy}{dx} = xy^3$

$\int \frac{1}{y^3}\,dy = \int x\,dx$ 1

$\frac{y^{-2}}{-2} = \frac{x^2}{2} + C$ 1 1

f(1) = 2 $\quad -\frac{1}{2y^2} = \frac{x^2}{2} + C$

$-\frac{1}{8} = \frac{1}{2} + C$

$-\frac{5}{8} = C$ 1

$-\frac{1}{2y^2} = \frac{x^2}{2} - \frac{5}{8}$

$\frac{4}{y^2} = -4x^2 + 5$

$y^2 = \frac{4}{-4x^2 + 5}$ 1

$\boxed{y = \sqrt{\frac{4}{-4x^2+5}}}$ since f(1) = 2 only positive $\sqrt{\ }$