2014 AB Calculus AB

Calculator

(1) $0 \le t \le 30$

$A(t) = 6.687(.931)^t$ amount of clippings

a) average rate of change $\frac{A(30)-A(0)}{30-0} = \frac{.782927 - 6.68}{30}$

$\frac{1}{30-0}\int_0^{30} A'(t)\,dt = \frac{1}{30}(A(30)-A(0))$

$= -.197$ pounds per day

b) $A'(15) = -.164$ lbs/day^2

The rate at which the clippings are decomposing is .164 lbs per day on day 15.

c) $A(t) = \frac{1}{30}\int_0^{30} A(t) = 2.753$

solve $A(t) = 2.753 \rightarrow t = 12.413$

d) $t = 30$ $A(30) = .78293$ $(30, .78293)$

$A'(30) = -.05598$

$L(t) = y - .783 = -.056(t-30)$

$L(Y) = .5 - .783 = -.056(t-30)$

$t = 35.054$ days

AB/BC

② Calalator

$f(x) = x^4 - 2.3x^3 + 4 \qquad y = 4$

a) $V = \pi \int_0^{2.3} (4 - -2)^2 - ((x^4 - 2.3x^3 + 4) - -2)^2 \, dx$

$\pi \int_0^{2.3} [36 - (x^4 - 2.3x^3 + 6)^2] \, dx$

$= \boxed{198.868}$

b) $V = \int_0^{2.3} \frac{1}{2} (4 - (x^4 - 2.3x^3 + 4))^2 \, dx$

$\frac{1}{2} \int_0^{2.3} (\cancel{4} - x^4 + 2.3x^3 - \cancel{4})^2 \, dx$

$= \boxed{3.574}$

c) $\int_0^k [4 - (x^4 - 2.3x^3 + 4)] \, dx = \int_k^{2.3} [4 - (x^4 - 2.3x^3 + 4)] \, dx$

or

$\int_0^k [4 - (x^4 - 2.3x^3 + 4)] \, dx = \frac{1}{2} \int_0^{2.3} (4 - f(x)) \, dx$

or

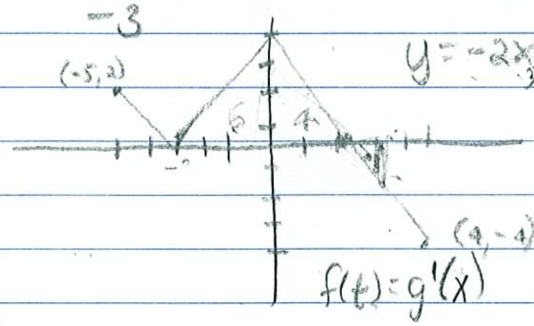
$\int_k^{2.3} (4 - f(x)) \, dx = \frac{1}{2} \int_0^{2.3} (4 - f(x)) \, dx$

AB

Calculator

③ $g(x) = \int_{-3}^{x} f(t)\,dt$ (f(t) = $g'(x)$) graph is $f = g'$

a) $g(3) = \int_{-3}^{3} f(t)\,dt = 6 + 4 - 1 = \boxed{9}$ ①



b) g inc where g' is positive ①
g ccd where g'' neg → g' decreasing
$\boxed{(-5, -3)\ (0, 2)}$ ①

c) $h(x) = \dfrac{g(x)}{5x}$

$h'(x) = \dfrac{5x\,g'(x) - g(x)5}{(5x)^2}$ ②

$h'(3) = \dfrac{15(-2) - 9(5)}{225} = \dfrac{-30-45}{225} = \dfrac{-75}{225}$ $\boxed{\dfrac{-1}{3}}$ ①

d) $p(x) = f(x^2 - x)$
$p'(x) = f'(x^2 - x)\ (2x - 1)$ ⑤
$p'(-1) = f'(1 - (-1))(-2 - 1)$
$= f'(2)(-3)$ f'(2) is slope at 2, -2
$= -2(-3) = \boxed{6}$ ①

AB/BC

(4) a) $\frac{V_A(8) - V_A(2)}{8-2} = \frac{-120-100}{8-2} = \frac{-220}{6}$

$\boxed{-110/3}$ m/min²

1

b) find is diff/cont.

2

Yes, since $V_A(t)$ is diff/cont, since $V_A(5) = 40$ and $V_A(8) = -120$, $V_A(t)$ must pass through -100. (IVT)

c) $300 + \int_2^{12} V_a(t)\,dt$

3

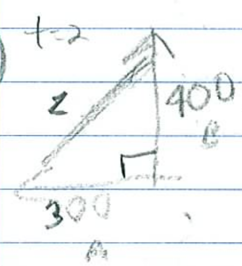
trapezoid sum

$= \frac{1}{2}(3)(100+40) + \frac{1}{2}(3)(40+-120) + \frac{1}{2}(4)(-120+-150)$

$= 210 + (-120) + (-540) = -450$

$300 + -450 = \boxed{-150}$

150 meters west of Station

d) 

Train B

$V_A(2) = -5(2)^2 + 60(2) + 25$

$= -20 + 120 + 25$

$= 125$

$x = 300$

$\frac{dx}{dt} = 100$

$y = 400$

$\frac{dy}{dt} = 125$

$z = 500$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$100(300) + 400(125) = 500\frac{dz}{dt}$

$300 + 500 = 5\frac{dz}{dt}$ $\quad \frac{800}{5}$

$\boxed{160 = \frac{dz}{dt}}$ m/min

AB/BC

5) a) rel min. where f' changes from negative to positive

$x = 1$

b) By Rolles Thm since f is cont/diff and $f'(1) = f'(-1)$, there is a place where $f''(x) = 0$

c) $h(x) = \ln(f(x))$

$h'(x) = \dfrac{f'(x)}{f(x)}$

$h'(3) = \dfrac{f'(3)}{f(3)} = \dfrac{1/2}{7} = \frac{1}{2} \cdot \frac{1}{7} = \boxed{1/14}$

d) $\int_{-2}^{3} f'(g(x))\, g'(x)\, dx$

$f(g(x))\Big|_{-2}^{3} = f(g(3)) - f(g(-2))$

$f(1) - f(-1)$

$2 - 8 = \boxed{-6}$

AB/BC

c) $\int_0^k [4-(x^4-2.3x^3+4)]\,dx = \int_k^{2.3} [4-(x^4-2.3x^3+4)]\,dx$

or

$\int_0^k [4-(x^4-2.3x^3+4)]\,dx = \frac{1}{2}\int_0^{2.3} (4-f(x))\,dx$

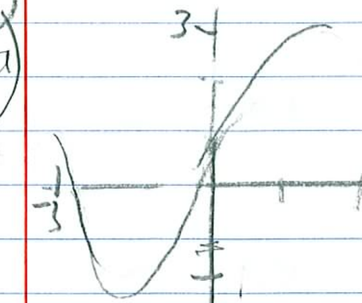
or

$\int_k^{2.3} (4-f(x))\,dx = \frac{1}{2}\int_0^{2.3} (4-f(x))\,dx$

AB

3) a) $g(x) = \int_{-3}^{x} f(t)\,dt$

6) a)



b) $(0,1)$ $\frac{dy}{dx} = (3-1)\cos(0)$

$dy/dx = 2(1) = 2$

$y-1 = 2(x-0)$ $y = 2x+1$

$y-1 = 2(.2)$

$y-1 = .4$ $y = 1.4$

c) $\frac{dy}{dx} = (3-y)\cos x$ $(0,1)$

$u = 3-y$, $du = -1$

$\int \frac{1}{3-y}\,dy = \int \cos x\,dx$

$-\ln|3-y| = \sin x + C$

$-\ln 2 = \sin(0) + C$

$-\ln 2 = C$

$-\ln|3-y| = \sin x - \ln 2$

$\ln|3-y| = -\sin x + \ln 2$

$|3-y| = e^{-\sin x + \ln 2}$

$|3-y| = e^{-\sin x} \cdot 2$

$3-y = \pm 2e^{-\sin x}$

$-y = \pm 2e^{-\sin x} - 3$

$y = -2e^{-\sin x} + 3$ or $y = 3 - e^{(\ln 2 - \sin x)}$