End of Practice Exam 2016

$\boxed{\text{key}}$ AB

$\boxed{\text{no calculator!}}$

1. $\int_{-1}^{3} (x^2 - 2x)\,dx =$

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$ (C) 8 (D) 12

$\frac{x^3}{3} - x^2 \Big|_{-1}^{3}$

$\left(\frac{27}{3} - 9\right) - \left(-\frac{1}{3} - 1\right)$

$0 - \left(-\frac{4}{3}\right) = 4/3$

2. If $f(x) = e^{2x}(x^3 + 1)$, then $f'(2) =$

(A) $6e^4$ (B) $21e^4$ (C) $24e^4$ (D) $30e^4$

$f'(x) = e^{2x}(3x^2) + (x^3+1)(2e^{2x})$

$f'(2) = e^4(12) + 9(2e^4) = 12e^4 + 18e^4 = 30e^4$

3. Let f be a differentiable function such that $f(2) = 4$ and $f'(2) = -\frac{1}{2}$. What is the approximation for $f(2.1)$ found by using the line tangent to the graph of f at $x = 2$?

(A) 2.95 (B) 3.95 (C) 4.05 (D) 4.1

$y - 4 = -\frac{1}{2}(x - 2)$

$y - 4 = -\frac{1}{2}(2.1 - 2)$

$y - 4 = -\frac{1}{2}(.1)$

$y = -.05 + 4 = 3.95$

4. Let g be the function defined by $g(x) = x^4 + 4x^3$. How many relative extrema does g have?

(A) Zero (B) One (C) Two (D) Three

$g'(x) = 4x^3 + 12x^2 = 0$

$4x^2(x+3) = 0$

$x = 0, -3$

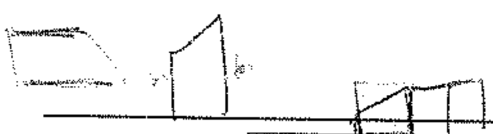
5. The velocity of a particle moving along the x-axis is given by $v(t) = 2 - t^2$ for time $t > 0$. What is the average velocity of the particle from time $t = 1$ to time $t = 3$?

(A) -4 (B) -3 (C) $-\frac{7}{3}$ (D) $\frac{7}{3}$

$\frac{1}{3-1}\int_1^3 (2 - t^2) = 2t - \frac{t^3}{3}\Big|_1^3$

$\frac{1}{2}\left[(6-9) - \left(2 - \frac{1}{3}\right)\right]$

$\frac{1}{2}\left(-3 - \frac{5}{3}\right) = \frac{1}{2}\left(-\frac{14}{3}\right) = -\frac{7}{3}$



$\frac{1}{2}h(b_1 + b_2)$

t (hours)	0	2	7	9
$R(t)$ (tons per hour)	15	9	5	4

$\frac{1}{2}(2)(15+9) + \frac{1}{2}(5)(9+5) + \frac{1}{2}(2)(5+4)$

$24 + 35 + 9 = 68$

6. On a certain day, the rate at which material is deposited at a recycling center is modeled by the function R, where $R(t)$ is measured in tons per hour and t is the number of hours since the center opened. Using a trapezoidal sum with the three subintervals indicated by the data in the table, what is the approximate number of tons of material deposited in the first 9 hours since the center opened?

(A) 68 (B) 70.5 (C) 85 (D) 136

Left sum 2(15) +
right sum 2(9)

add plural

7. What is the total area of the regions between the curves $y = 6x^2 - 18x$ and $y = -6x$ from $x = 1$ to $x = 3$?

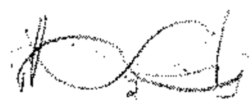
(A) 4 (B) 12 (C) 16 (D) 20

$6x^2 - 18x = -6x$

$6x^2 - 12x = 0$

$6x(x-2) = 0$

$x = 0, 2$



$\int_1^2 -6x - (6x^2 - 18x) + \int_2^3 (6x^2 - 18x) - (-6x)$

$-6x^2 + 12x \qquad 6x^2 - 12x$

$-2x^3 + 6x^2 \Big|_1^2 \qquad 2x^3 - 6x^2 \Big|_2^3$

$= 4 \qquad (54 - 54) - (16 - 24) = 8$

$4 + 8 = \boxed{12}$

8. The function g is defined by $g(x) = x^2 + bx$, where b is a constant. If the line tangent to the graph of g at $x = -1$ is parallel to the line that contains the points $(0, -2)$ and $(3, 4)$, what is the value of b ?

(A) -1 ~~(B) [illegible]~~ (C) $\frac{3}{2}$ (D) 4

$\frac{4-(-2)}{3-0} = 2$

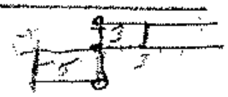
$g'(x) = 2x + b$

$2(-1) + b = 2$

$-2 + b = 2$

$b = 4$

$f(x) = \begin{cases} \frac{|x|}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$

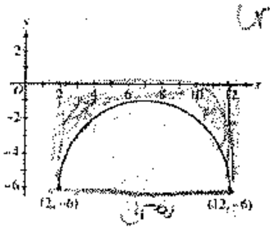


9. The function f is defined above. The value of $\int_{-5}^{3} f(x)\,dx$ is

(A) -2 (B) 2 (C) 8 (D) nonexistent

$-5 + 3$

$\int_{-5}^{0} \qquad + \int_{0}^{3}$



Graph of f

under x

rectangle − semicircle

$-\left(10(6) - \frac{1}{2}\pi(5^2)\right)$

$-\left(60 - \frac{25}{2}\pi\right)$

$-60 + \frac{25}{2}\pi$

11. The graph of $y = f(x)$ consists of a semicircle with endpoints at $(2, -6)$ and $(12, -6)$, as shown in the figure above. What is the value of $\int_2^{12} f(x)\,dx$?

(A) $-\frac{25\pi}{2}$ (B) $\frac{25\pi}{2}$ (C) $60 + \frac{25\pi}{2}$ (D) $60 - \frac{25\pi}{2}$

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12. An object moves along a straight line so that at any time t its acceleration is given by $a(t) = 6t$. At time $t = 0$, the object's velocity is 10 and the object's position is 7. What is the object's position at time $t = 2$?

(A) 12 (B) 27 (C) 28 (D) 35

$v(t) = \int 6t = 3t^2 + C$

$3(0) + C = 10$

$C = 10$

$v(t) = 3t^2 + 10$

$x(t) = \int 3t^2 + 10 = t^3 + 10t + C = 7$

$C = 7$

$x(t) = t^3 + 10t + 7$

$x(2) = 8 + 20 + 7 = 35$

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13. If $y = \cos x - \ln(2x)$, then $\frac{d^3y}{dx^3} =$

(A) $\sin x - \frac{2}{x^3}$

(B) $-\sin x - \frac{2}{x^3}$

(C) $\sin x - \frac{1}{x^3}$

(D) $-\sin x - \frac{1}{x^3}$

$y' = -\sin x - \frac{2}{2x} = -\sin x - \frac{1}{x}$

$y'' = -\cos x + x^{-2}$

$y''' = \sin x - 2x^{-3}$

$\sin x - \frac{2}{x^3}$

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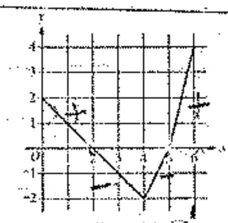
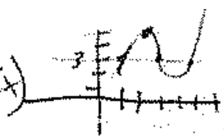
g inc...

g dec...



Graph of f

g(x)



$\int |v(t)|$

14. The graph of the function f, shown above, consists of three line segments. If the function g is an antiderivative of f such that $g(2) = 5$, for how many values of c, where $0 \le c \le 6$, does $g(c) = 3$?

(A) Zero (B) One (C) Two (D) Three

$$f(x) = \begin{cases} 6 + cx, & \text{for } x \le 1 \\ 9 + 2\ln x, & \text{for } x \ge 1 \end{cases}$$

$6 + c$

9

$6 + c = 9$

$c = 3$

15. Let f be the function defined above, where c is a constant. If f is continuous at $x = 1$, what is the value of c ?

(A) 2 (B) 3 (C) 5 (D) 9

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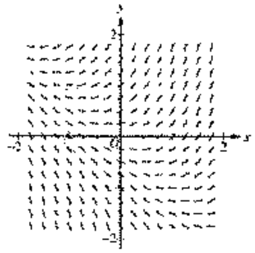
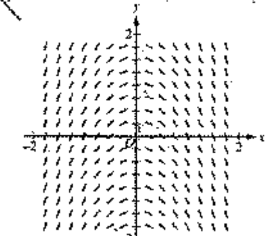
16. Which of the following could be a slope field for the differential equation $\frac{dy}{dx} = x^2 + y$?

(A)

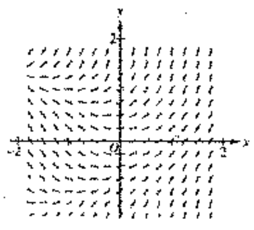


(B)

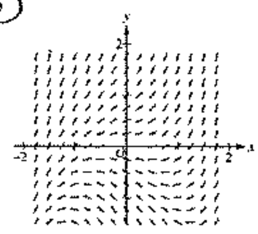


Check points in $\frac{dy}{dx}$ for slopes going correct directions

(C)



(D)



17. The function $y = e^{3x} - 5x + 7$ is a solution to which of the following differential equations?

(A) $y'' - 3y' - 15 = 0$

(B) $y'' - 3y' + 15 = 0$

(C) $y'' - y' - 5 = 0$

(D) $y'' - y' + 5 = 0$

$y' = 3e^{3x} - 5$
$y'' = 9e^{3x}$
a) $9e^{3x} - 3(3e^{3x} - 5) - 15 = 0$
$9e^{3x} - 9e^{3x} + 15 - 15 = 0$ ✓

$\sin^{-1} x$

18. If $f(x) = \sin^{-1} x$, then $f'\left(\frac{\sqrt{3}}{2}\right) =$

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{4}{7}$ (D) 2

$f'(x) = \frac{1}{\sqrt{1-x^2}}$

$f'\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{\sqrt{1-\frac{3}{4}}} = \frac{1}{\sqrt{\frac{1}{4}}} = \frac{1}{\frac{1}{2}} = 2$

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19. $\lim\limits_{x \to e} \frac{(x^{20} - 3x) - (e^{20} - 3e)}{x - e}$ is

(A) 0 (B) $20e^{19} - 3$ (C) $e^{20} - 3e$ (D) nonexistent

$\frac{e^{20} - 3e - e^{20} + 3e}{e - e} = \frac{0}{0}$ L'hopital

$\lim\limits_{x \to e} \frac{20x^{19} - 3}{1} = 20e^{19} - 3$

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20. Let $y = f(x)$ be a twice-differentiable function such that $f(1) = 2$ and $\frac{dy}{dx} = y^3 + 3$. What is the value of $\frac{d^2y}{dx^2}$ at $x = 1$?

(A) 12 (B) 60 (C) 132 (D) 165

$3y^2 \boxed{\frac{dy}{dx}}$
$3(2)^2(2^3 + 3) = 132$
$12(11)$

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x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
−2	−6	9	−10	16
1	5	−3	3	−2
3	0	7	8	3

21. The table above gives values of f, f', g, and g' for selected values of x. If $h(x) = f(g(x))$, what is the value of $h'(1)$?

(A) −19 (B) 14 (C) 7 (D) 9

$h'(x) = f'(g(x))\, g'(x)$
$h'(1) = f'(g(1))\, g'(1)$
$= f'(3)\ (-2)$
$7(-2) = -14$

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22. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$ with the initial condition $f(0) = -2$. Which of the following is an expression for $f(x)$?

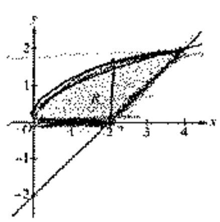
(A) $-2 + \sqrt{x^2 + 2x}$

(B) $2 + \sqrt{x^2 + 2x}$

(C) $-\sqrt{x^2 + 2x + 4}$

(D) $\sqrt{x^2 + 2x + 4}$

$\int y\,dy = \int x + 1\,dx$
$\frac{y^2}{2} = \frac{x^2}{2} + x + C$
$2 = C$
$\frac{y^2}{2} = \frac{x^2}{2} + x + 2$
$y^2 = x^2 + 2x + 4$
$y = \pm\sqrt{x^2 + 2x + 4}$
$y = -\sqrt{x^2 + 2x + 4}$



Split at 2

$\pi\int_0^2 \sqrt{x}^2\,dx + \pi\int_2^4 (\sqrt{x})^2 - (x-2)^2\,dx$

$\pi\int_0^2 x\,dx + \pi\int_2^4 x - (x-2)^2$

23. Let R be the shaded region bounded by the graph of $y = \sqrt{x}$, the graph of $y = x - 2$, and the x-axis, as shown in the figure above. Which of the following gives the volume of the solid generated when R is revolved about the x-axis?

(A) $\pi\int_0^4 \left(x - (x-2)^2\right) dx$

(B) $\pi\int_0^4 (\sqrt{x} - (x-2))^2\,dx$

(C) $\pi\int_0^2 x\,dx + \pi\int_2^4 \left(x - (x-2)^2\right) dx$

(D) $\pi\int_0^2 x\,dx + \pi\int_2^4 (\sqrt{x} - (x-2))^2\,dx$

$\frac{\tan(3-3)}{3e^{3-3} - 3} = \frac{\tan 0}{3-3} = \frac{0}{0}$ L'hopital

24. $\lim\limits_{x \to 3} \frac{\tan(x-3)}{3e^{x-3} - x}$ is

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) nonexistent

$\lim\limits_{x \to 3} \frac{\sec^2(x-3)}{3e^{x-3} - 1} = \frac{\sec^2(0)}{3e^0 - 1} = \frac{1}{2}$

25. Let f be a function with first derivative defined by $f'(x) = \frac{3x^2 - 6}{x^2}$ for $x > 0$. It is known that $f(1) = 9$ and $f(3) = 11$. What value of x in the open interval $(1, 3)$ satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[1, 3]$?

(A) $\sqrt{6}$ (B) $\sqrt{3}$ (C) $\sqrt{2}$ (D) 1

$f(1) = 9$
$f(3) = 11$

$f'(x) = \frac{f(b) - f(a)}{b - a}$

$\frac{3x^2 - 6}{x^2} = \frac{11 - 9}{3 - 1}$

$\frac{3x^2 - 6}{x^2} = 1$

$3x^2 - 6 = x^2$

$2x^2 = 6$

$x^2 = 3$ $x = \sqrt{3}$

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26. $\int_1^2 \frac{x^2 - x - 5}{x + 2}\, dx =$

(A) $-\frac{3}{2} + \ln\frac{4}{3}$ (B) $-\frac{25}{21}$ (C) $\frac{5}{2} + 3\ln\frac{3}{4}$ (D) $\frac{23}{45}$

$x + 2 \overline{\big) x^2 - x - 5}$ quotient $x - 3 + \frac{1}{x+2}$

$-x^2 + 2x$

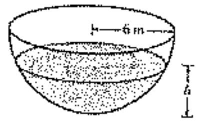
$-3x - 5$

$-3x - 6$

$\int x - 3 + \frac{1}{x+2}$

$\frac{x^2}{2} - 3x + \ln|x+2| \Big|_1^2 = (2 - 6 + \ln 4) - (\frac{1}{2} - 3 + \ln 3)$

$-4 + \ln 4 + \frac{5}{2} - \ln 3$ → $-\frac{3}{2} + \ln\frac{4}{3}$



$r = 6$
$h = 3$
$\frac{dh}{dt} = -\frac{1}{2}$
$\frac{dA}{dt} = ?$

$A = 12\pi h - \pi h^2$

$\frac{dA}{dt} = 12\pi \frac{dh}{dt} - 2\pi h \frac{dh}{dt}$

$= 12\pi(-\frac{1}{2}) - 2\pi(3)(-\frac{1}{2})$

$-6\pi + 3\pi$

-3π

27. A hemispherical water tank, shown above, has a radius of 6 meters and is losing water. The area of the surface of the water is $A = 12\pi h - \pi h^2$ square meters, where h is the depth, in meters, of the water in the tank. When $h = 3$ meters, the depth of the water is decreasing at a rate of $\frac{1}{2}$ meter per minute. At that instant, what is the rate at which the area of the water's surface is decreasing with respect to time?

(A) 3π square meters per minute
(B) 6π square meters per minute
(C) 9π square meters per minute
(D) 27π square meters per minute

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28. Consider a triangle in the xy-plane. Two vertices of the triangle are on the x-axis at $(1, 0)$ and $(5, 0)$, and a third vertex is on the graph of $y = \ln(2x) - \frac{1}{2}x + 5$ for $\frac{1}{2} \le x \le 8$. What is the maximum area of such a triangle?

(A) $\frac{19}{2}$
(B) $2\ln 2 + 9$
(C) $2\ln 4 + 8$
(D) $2\ln 16 + 2$

$A = \frac{1}{2}(4)(\ln(2x) - \frac{1}{2}x + 5)$

$A = 2\ln(2x) - x + 10$

$A' = 2(\frac{2}{2x}) - 1 = \frac{2}{x} - 1 = 0$

$\frac{2}{x} = 1$

$A(1) = 2\ln 2 + 9$

$A(2) = 2\ln 4 + 8$

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29. The function f is defined by $f(x) = x^3 + 4x + 2$. If g is the inverse function of f and $g(2) = 0$, what is the value of $g'(2)$?

$f'(x) = 3x^2 + 4$ $f'(0) = 4$ $f(0) = 2$

$g'(x) = \frac{1}{f'(g(x))}$ $= \frac{1}{f'(0)} = \frac{1}{4}$

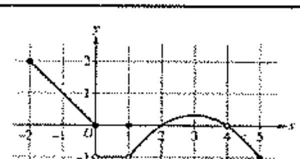
(A) $-\frac{1}{16}$ (B) $-\frac{4}{81}$ (C) $\frac{1}{4}$ (D) 1

$\boxed{\text{Calculator}}$

76. To help restore a beach, sand is being added to the beach at a rate of $s(t) = 65 + 24\sin(0.3t)$ tons per hour, where t is measured in hours since 5:00 A.M. How many tons of sand are added to the beach over the 3-hour period from 7:00 A.M. to 10:00 A.M.?

(A) 255.368 (B) 223.271 (C) 85.123 (D) 10.388

$\int_2^5 65 + 24\sin(.3t)\,dt = 255.368$



Graph of f

77. The graph of the function f is shown above. For what values of a does $\lim_{x \to a} f(x) = 0$?

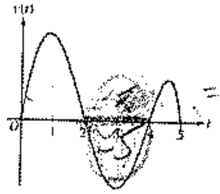
(A) 2 only
(B) 2 and 4
(C) 0 and 2 only
(D) 0, 1, and 2

78. The second derivative of a function f is given by $f''(x) = \sin(3x) - \cos(x^2)$. How many points of inflection does the graph of f have on the interval $0 < x < 3$?

(A) One (B) Three (C) Four (D) Five

$\sin(3x) - \cos(x^2) = 0$

change from +/−



$= 13, 10$

$\int_2^4 |v(t)| = 13$

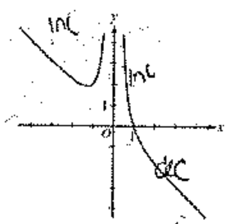
79. Over the time interval $0 \le t \le 5$, a particle moves along the x-axis. The graph of the particle's velocity, v, is shown above. Over the time interval $0 \le t \le 5$, the particle's displacement is 3 and the particle travels a total distance of 13. What is the value of $\int_2^4 v(t)\,dt$?

Change in position

(A) −10 (B) −5 (C) 5 (D) 10

80. The temperature in a room at midnight is 20 degrees Celsius. Over the next 24 hours, the temperature changes at a rate modeled by the differentiable function H, where $H(t)$ is measured in degrees Celsius per hour and time t is measured in hours since midnight. Which of the following is the best interpretation of $\int_0^6 H(t)\,dt$?

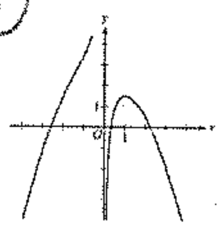
(A) The temperature of the room, in degrees Celsius, at 6:00 A.M.
(B) The average temperature of the room, in degrees Celsius, between midnight and 6:00 A.M.
(C) The change in the temperature of the room, in degrees Celsius, between midnight and 6:00 A.M.
(D) The rate at which the temperature in the room is changing, in degrees Celsius per hour, at 6:00 A.M.



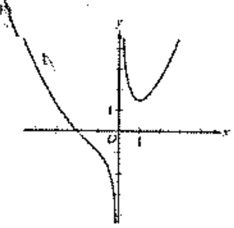
Graph of f'

81. The graph of f', the derivative of the function f, is shown above. Which of the following could be the graph

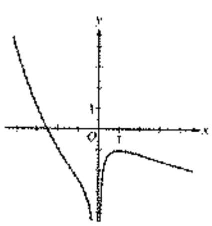
(A)



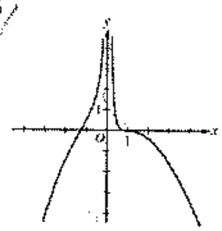
(B)



(C)



(D)

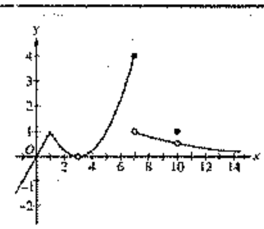


82. Let f be the function with derivative given by $f'(x) = \sin(x^2 - 3)$. At what values of x in the interval $-3 < x < 3$ does f have a relative maximum?

f' min

f' changes from + to −

(A) 1.732 and 2.478 only
(B) ~~2.478 and 1.732 only~~
(C) ~~−2.138, 0, and 2.138~~
(D) −2.478, −1.732, 1.732, and 2.478



Graph of f

83. The graph of the function f is shown above. At what value of x does f have a jump discontinuity?

(A) 1 (B) 3 (C) 7 (D) 10

84. Let f be a differentiable function such that $f(1) = \pi$ and $f'(x) = \sqrt{x^3 + 6}$. What is the value of $f(5)$?

(A) 11.941 (B) 14.587 (C) 24.672 (D) 27.814

$\int_1^5 \sqrt{x^3+6} = f(5) - f(1)$

$24.672 = f(5) - \pi$

$f(5) = 27.84$

85. People are entering a building at a rate modeled by $f(t)$ people per hour and exiting the building at a rate modeled by $g(t)$ people per hour, where t is measured in hours. The functions f and g are nonnegative and differentiable for all times t. Which of the following inequalities indicates that the rate of change of the number of people in the building is increasing at time t ?

(A) $f(t) > 0$

(B) $f'(t) > 0$

(C) $f(t) - g(t) > 0$

(D) $f'(t) - g'(t) > 0$

Increases when deriv is > 0

$f'(t) > g'(t)$

$f'(t) - g'(t) > 0$

86. The velocity of a particle moving along the x-axis is given by $v(t) = \sqrt{t} - \cos(e^t)$ for $t \geq 0$. Which of the following statements describes the motion of the particle at $t = 1$?

(A) The particle is moving to the left with positive acceleration.

(B) The particle is moving to the right with positive acceleration.

(C) The particle is moving to the left with negative acceleration.

(D) The particle is moving to the right with negative acceleration.

$v(1) = 1 - \cos(e) = 1.912$ moving right

$a(1) = 1.617$ (pos)

87. A tire that is leaking air has an initial air pressure of 30 pounds per square inch (psi). The function $t = f(p)$ models the amount of time t, in hours, it takes for the air pressure of the tire to reach p psi. What are the units for $f'(p)$?

(A) hours (B) psi (C) psi per hour (D) hours per psi

$t = f(p)$ time

$f'(p)$

88. The first derivative of the function f is defined by $f'(x) = \frac{x + 2e^{-x}}{x^2 + 0.7}$. On what intervals is f increasing?

(A) $-1.384 < x < -0.264$ only

(B) $x < -0.633$ and $x > 0.319$ only

(C) $-\infty < x < \infty$

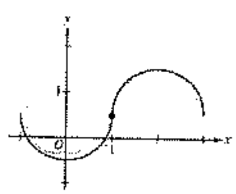
(D) There are no intervals on which f is increasing.

f increases when f' positive

x	0	4	6	8	13
$f(x)$	3	4.5	3	2.5	4.4

89. The table above shows selected values of a continuous function f. For $0 \leq x \leq 13$, what is the fewest possible number of times $f(x) = 4$?

(A) One (B) Two (C) Three (D) Four



Graph of h'

$\int^3 h'(x) = h(3) - h($

90. The function h is defined on the closed interval $[-1, 3]$. The graph of h', the derivative of h, is shown above. The graph consists of two semicircles with a common endpoint at $x = 1$. Which of the following statements about h must be true?

I. $h(-1) = h(3)$

II. h is continuous at $x = 1$. (deriv exists at 1, so must be continuous)

III. The graph of h has a vertical asymptote at $x = 1$.

(A) None (B) II only (C) I and II only (D) I and III only