AB MC/FR

All integration Techniques and Differential Equations
no calculator

1) $f'(x) = \frac{5}{3}\cos\left(\frac{1}{3}x\right) - 2\sin\left(\frac{2}{3}x\right)$

$f(x) = 5\sin\left(\frac{1}{3}x\right) + 3\cos\left(\frac{2}{3}x\right) + C$

$f(\pi/2) = 5/2 + 3/2 + C = 7$

$4 + C = 7 \qquad C = 3$

$f(\pi) = 5\sqrt{3}/2 - 3/2 + 3 \quad = \frac{5\sqrt{3}}{2} + \frac{3}{2}$ (A)

2) $\frac{dy}{dx} = 7 + 5\tan^2 x \quad f(0) = 4$

$\int 7 + 5\int(\sec^2 x - 1) = 7x + 5\tan x - 5x + C = 4$

$C = 9$

$2x + 5\tan x - 5 + 9 = 2x + 5\tan x + 4$ (B)

3) $\int (2-x)^2 + \frac{2}{(2-x)^2}$

$\int 4 - 4x + x^2 + \int \frac{2}{(2-x)^2} \quad u = 2-x \quad du = -1$

$-2\int u^{-2} + u^{-1} \quad \frac{1}{u} \quad \int \frac{1}{u^2}$

$4x - 2x^2 + \frac{x^3}{3} + \frac{2}{2-x} + C = 0 \qquad C = 0$

$F(3) = 12 - 18 + 9 - 2 = 1$ (E)

5) $F(8) - F(2) = 14$ $\qquad F(2) = \frac{1}{2}(2)(2) = 2$

$F(8) = 16$ $\qquad F(1) = \frac{1}{2}(1)(1) = 1/2$

$F(1) = 1/2$

$F(8) - F(1) = 15\frac{1}{2}$ (E)

4) $f'(x) = 2e^{2x} \qquad f(\ln 2) = 2 \qquad f(0) =$

$\int 2e^{2x} = \cancel{2}\left(\frac{1}{2}\right)e^{2x} + C$

$e^{2\ln 2} + C = 2 \qquad C = -2$

$f(x) = e^{2x} - 2 \qquad f(0) = 1 - 2 = -1$ (A)

6) $\frac{dy}{dx} = x\sqrt{2x^2+1}$ $(2, 4)$

$\int dy = \int x\sqrt{2x^2+1}\, dx$ $u = 2x^2+1$ $du = 4x$

$y = \frac{1}{4}\int \sqrt{u}\, du$

$y = \frac{1}{4} \cdot \frac{2}{3} u^{3/2} + C$

$y = \frac{1}{6}(2x^2+1)^{3/2} + C$

$4 = \frac{27}{6}\frac{9}{2} + C$ $C = -1/2$

$y = \frac{1}{6}(2x^2+1)^{3/2} - 1/2$

y-int → x=0 $\frac{1}{6} - \frac{1}{2} = -1/3$ C

7) $\int_1^4 \frac{3}{\sqrt{x}(\sqrt{x}+2)^2}\, dx$ $u = \sqrt{x}+2$

$du = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$6\int \frac{du}{u^2} = \frac{-6}{u} = \frac{-6}{\sqrt{x}+2}\Big|_1^4 = \frac{-3}{2} - (-2)$ $\frac{-3}{2}+2 =$

$\frac{1}{2}$ B

8) $\int_0^1 \frac{8+x}{1+x^2}\, dx = \int_0^1 \frac{8}{1+x^2} + \int_0^1 \frac{x}{1+x^2}$ $u = 1+x^2$ $du = 2x$

$a=1$ $u=x$ $du=1$

$8 \arctan x\Big|_0^1 + \frac{1}{2}\ln|1+x^2|\Big|_0^1$

$8(\arctan 1 - \arctan 0) + \frac{1}{2}(\ln 2 - \ln 1)$

$= 8(\pi/4 - 0) + \frac{1}{2}(\ln 2 - 0)$

$= 2\pi + \frac{1}{2}\ln 2 = \frac{1}{2}(4\pi + \ln 2)$ A

9) $y = Pe^{rt}$ $(0, 1000)$ $(7, 1200)$ $(12, y)$

$1000 = Pe^{r(0)}$

$\boxed{P = 1000}$

$1200 = 1000e^{r(7)}$ $1.2 = e^{7r}$

$\ln 1.2 = 7r$ $\boxed{r = .026}$

$y = 1000\, e^{.026(12)}$

$y = 1366.15$ C

10) $\frac{dy}{dx} = 2y^2$ $y = -1, x = 1$ $x = 2, y =$

$\frac{1}{y^2} dy = 2dx$

$\frac{-1}{y} = 2x + C$ $1 = 2 + C$ $C = -1$

$-\frac{1}{y} = 2x - 1$ $-\frac{1}{y} = 4 - 1$ $-\frac{1}{y} = 3$

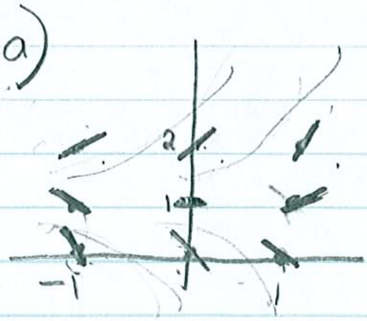
$y = -1/3$ B

11) C

12) 2007B #5

$\frac{dy}{dx} = \frac{1}{2}x + y - 1$

a)



2 points

b) $\frac{d^2y}{dx^2} = \frac{1}{2} + \frac{dy}{dx}$ (1)

$\frac{d^2y}{dx^2} = \frac{1}{2} + \frac{1}{2}x + y - 1$ $\boxed{2pts}$

$\frac{d^2y}{dx^2} = \frac{1}{2}x + y - \frac{1}{2}$

$y = -\frac{x}{2} + \frac{1}{2}$

solution curves ccu above $y = -\frac{x}{2} + \frac{1}{2}$ 1pt