AP Review MC 1-20 Sat. 4/7 2018

① $\lim_{x\to 1} f(g(x)) \to \lim_{x\to 1} f(g(1)) \to \lim_{x\to 1} f(2) \to 3$ C

② $\lim_{x\to 0} \frac{7x-\sin x}{x^2+\sin(3x)} \to \frac{0}{0}$ indeterminate form

L'hopitals $\lim_{x\to 0} \frac{7-\cos x}{2x+3\cos(3x)} = \frac{7-1}{3} = 2$ B

③ $f(x) = \sin(\ln(2x))$

$f'(x) = \cos(\ln(2x))\left(\frac{2}{2x}\right) \to \frac{\cos(\ln(2x))}{x}$ B

④ I f'' II f III f' C

⑤ $y = 4x+1$ $y' = 4$

$g(1/2) + g'(1/2)$

$3 + 4 = 7$ D

⑥ $v(t) = (t-5)(t-2)^2$

$a(t) = (t-5)\,2(t-2) + (t-2)^2(1) = 0$

$(t-2)[(t-5)(2) + (t-2)]$

$(t-2)[2t-10+t-2] = (t-2)(3t-12) = 0$

2, 4 C

⑦ $C''(500) = 80$ D

⑧ $\lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{1+\frac{3k}{n}} \cdot \frac{1}{n}$ A

⑨ $f(x) = \begin{cases} x & x < 2 \\ 3 & x \geq 2 \end{cases}$ $\int_{-1}^{4} f(x)\,dx = \int_{-1}^{2} f(x) + \int_{2}^{4} f(x)$

$\int_{-1}^{2} x + \int_{2}^{4} 3$

$\frac{x^2}{2}\Big|_{-1}^{2} + 3x\Big|_{2}^{4}$ $(2 - 1/2) + (12-6)$

$3/2 + 6 = 15/2$ B

⑩ $\int e^x \cos(e^x+1)\,dx$ $\quad u = e^x+1$

$\int \cos u\,du$ $\quad du = e^x dx$

$\sin u + C$

$\sin(e^x+1) + C$ (A)

⑪ $5e^{.2t} + 4t$

$\int_0^{10} 5e^{.2t} + 4t\,dt$ $\quad u = .2t$, $du = .2$

$(5)\left(5e^{.2t}\Big|_0^{10}\right) + 2t^2\Big|_0^{10}$

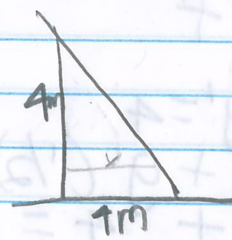
$5(5e^2 - 5) + 200$

$25e^2 - 25 + 200 = 25e^2 + 175$ (C)

no ⑫ $D(x) = \sqrt{x+1}$

$\int (4-x)\,D(x)\,dx$

(D)



⑬ $\frac{dy}{dx} = y\sec^2 x$ $\quad y(\pi/4) = -1$

$\int \frac{1}{y}\,dy = \int \sec^2 x\,dx$

$\ln|y| = \tan x + C \longleftarrow$

$\ln|-1| = \tan(\pi/4) + C$

$0 = 1 + C \quad C = -1$

$\ln|y| = \tan x - 1$ $\quad e^{\tan x - 1}$

$|y| = e^{\tan x} \cdot e^{-1}$

$y = \pm e^{\tan x} \cdot e^{-1}$ $\quad e^1 \cdot e^{-1}$

$y = -e^{\tan x} \cdot e^{-1}$

$y = -e^{\tan x - 1}$ (B)

14) $(-4, 4)$ discontinuous
$x = -1, 1, 2$ C

15) $g'(2)$ $\boxed{g'(x) = \frac{1}{f'(g(x))}}$

$g'(2) = \frac{1}{f'(g(2))} = \frac{1}{f'(1)} = \frac{1}{-5}$ A

$g(2, 1)$
$f(1, 2)$

derivatives of inverses are reciprocals

16) $f'(x) = -\frac{x}{3} + \cos(x^2)$ $0 < x \leq 3$
$f'(x) = 0$, changes from $-$ to pos
find intersection of $f'(x)$ and x-axis
$x = 2.372$ C

17) $g''(x) = 2^{-x^2} + \cos x + x$ $-5 < x < 5$
$g(x)$ concave up $g''(x)$ positive $-1.016 < x < 5$)
B

18) $H(t) = 55 - 9\cos\left(\frac{2\pi}{365}(t+10)\right)$ $t=0$
instantaneous rate of change at $t = 90$
$H'(90)$ on calculator $= .153°$ B

19) $h(x) = f(2x)$
(i) no way of knowing if $h'(x)$ is pos or neg between chart values (eliminate B & D)
(ii) no need to check
(iii) $h'(x) = f'(2x)(2) = 3$ $x=0$ $f'(0)(2) = 0$
$x=2$ $f'(4)(2) = 2$
$f'(2x) = 3/2$ in between C

(20) $h(x) = \frac{1}{\sqrt{x^5+1}}$ $g(2)=3$

$g(x) = \int_2^4 \frac{1}{\sqrt{x^5+1}} = .152 + 3 = 3.152$

D