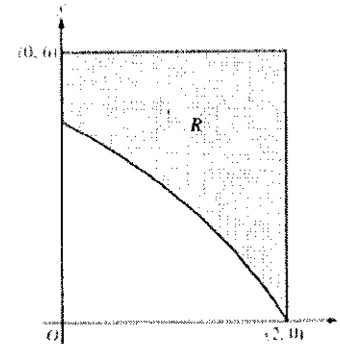
Ch 6/7 FR – More Practice

1. CALC

In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.



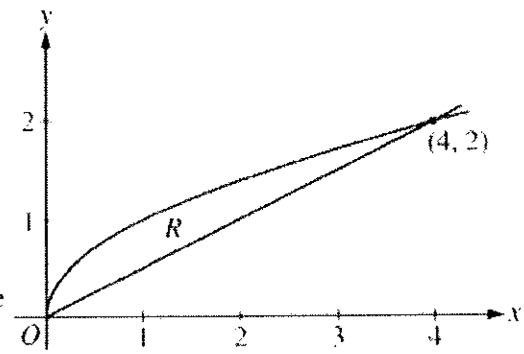
(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

2. NO CALC

Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.



(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Find the volume of this solid.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

3. CALC

Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

4. NO CALC

Let R be the region in the first quadrant enclosed by the graph of $y = \sqrt{6x + 4}$, the line $y = 2x$, and the y-axis.

(a) Find the area of R.

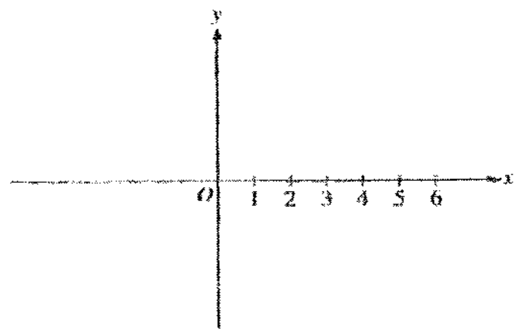
(b) Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the <u>x-axis</u>.

(c) Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the <u>y-axis</u>.

5. no calc

Let f be the function given by $f(x) = \sqrt{x-3}$.

(a) On the axes provided below, sketch the graph of f and shade the region R enclosed by the graph of f, the x-axis, and the vertical line $x = 6$.



(b) Find the area of the region R described in part (a).

(c) Rather than using the line $x = 6$ as in part (a), consider the line $x = w$, where w can be any number greater than 3. Let $A(w)$ be the area of the region enclosed by the graph of f, the x-axis, and the vertical line $x = w$. Write an integral expression for $A(w)$.

(d) Let $A(w)$ be as described in part (c). Find the rate of change of A with respect to w when $w = 6$.

6.

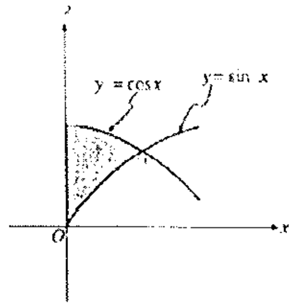
Let f be the function given by $f(x) = e^{-x}$, and let g be the function given by $g(x) = kx$, where k is the nonzero constant such that the graph of f is tangent to the graph of g.

(a) Find the x-coordinate of the point of tangency and the value of k.

(b) Let R be the region enclosed by the y-axis and the graphs of f and g. Using the results found in part (a), determine the area of R.

(c) Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid generated by revolving the region R, given in part (b), about the <u>x-axis</u>.

7. no calc



Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = \sin x$ and $y = \cos x$, as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Find the volume of the solid whose base is R and whose cross sections cut by planes perpendicular to the x-axis are squares.

8. no calc

Let R be the region enclosed by the graphs of $y = e^x$, $y = (x-1)^2$, and the line $x = 1$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

9.

Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

(a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

(b) Terminal velocity is defined as $\lim_{t \to \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

(c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

10.

Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

(a) Find $f''(3)$.

(b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

11.

Consider the differential equation $\frac{dy}{dx} = \frac{3 - x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.