EXAM I
CALCULUS BC
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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1. $\displaystyle\int_0^5 \frac{dx}{\sqrt{3x+1}} =$

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) 1

(D) 2

(E) 6

Ans

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2. Which of the following is continuous at $x = 0$?

I. $f(x) = |x|$

II. $f(x) = e^x$

III. $f(x) = \ln(e^x - 1)$

(A) I only (B) II only (C) I and II only (D) II and III only (E) none

Ans

3. For what value of c will $x^2 + \frac{c}{x}$ have a relative minimum at $x = -1$?

(A) –4 (B) –2 (C) 2 (D) 4 (E) None of these

Ans

4. Which of the following could be a solution to the differential equation represented by the slope field at the right?

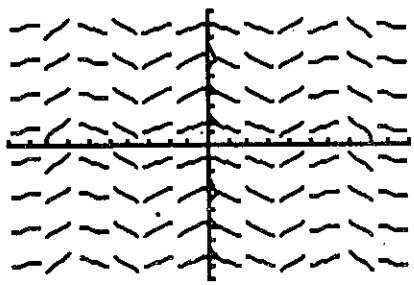
(A) $y = x^2$

(B) $y = \sin x$

(C) $y = \cos x$

(D) $y = e^x$

(E) $y = \ln x$



Ans

5. The volume of the solid generated by rotating about the x-axis the region enclosed between the curve $y = 3x^2$ and the line $y = 6x$ is given by

(A) $\pi \int_0^3 (6x - 3x^2)^2 \, dx$

(B) $\pi \int_0^2 (6x - 3x^2)^2 \, dx$

(C) $\pi \int_0^2 (9x^4 - 36x^2) \, dx$

(D) $\pi \int_0^2 (36x^2 - 9x^4) \, dx$

(E) $\pi \int_0^2 (6x - 3x^2) \, dx$

Ans

6. The asymptotes of the graph of the parametric equations $x = \frac{1}{t-1}$, $y = \frac{2}{t}$ are:

(A) $x = 1,\ y = 0$
(B) $y = 2$ only
(C) $x = -1,\ y = 2$
(D) $x = -1$ only
(E) $x = 0,\ y = -1$

Ans

7. $\int_1^{\infty} \frac{3x^2}{(1 + x^3)^2}\, dx =$

(A) $-\frac{1}{2}$
(B) 0
(C) $\frac{1}{2}$
(D) 1
(E) nonexistent

Ans

8. The radius of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^n}{n \cdot 3^n}$ is

(A) 3
(B) 2
(C) 1
(D) 0
(E) ∞

Ans

9. There is a point between $P(1, 0)$ and $Q(e, 1)$ on the graph of $y = \ln x$ such that the tangent to the graph at that point is parallel to the line through points P and Q. The x-coordinate of this point is

(A) $e - 1$

(B) e

(C) -1

(D) $\frac{1}{e - 1}$

(E) $\frac{1}{e + 1}$

Ans

10. If $4x^2 + 2xy + 3y = 9$, then the value of $\frac{dy}{dx}$ at the point $(2, -1)$ is

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) 2

(D) -2

(E) none of these

Ans

11. If $\frac{dy}{dx} = \sqrt{x}$, then the average rate of change of y with respect to x on the closed interval $[0, 4]$ is

(A) $\frac{1}{16}$ (B) 1 (C) $\frac{4}{3}$ (D) $\sqrt{2}$ (E) 2

Ans

12. A particle moves along the x-axis and its position for time $t \geq 0$ is $x(t) = \cos(2t) + \sec t$ When $t = \pi$, the acceleration of the particle is

(A) -6

(B) -5

(C) -4

(D) -3

(E) none of these

Ans

13. The region bounded by the x-axis and the part of the graph of $y = \sin x$ between $x = 0$ and $x = \pi$ is separated into two regions by the line $x = p$. If the area of the region for $0 \leq x \leq p$ exceeds the area of the region for $p \leq x \leq \pi$ by one square unit, then $p =$

(A) $\arccos \frac{1}{4}$

(B) $\arccos \frac{1}{3}$

(C) $\frac{\pi}{4}$

(D $\frac{\pi}{3}$

(E) $\frac{2\pi}{3}$

Ans

14. The graph of $y = 5x^4 + 3x^5$ has a point of inflection at

(A) (0, 0) only

(B) (1, 8) only

(C) (–1, 2) only

(D) (0, 0) and (1, 8)

(E) (0,0) and (–1, 2)

Ans

15. If $h(x) = [f(x)]^2 + f(x)g(x)$, $f'(x) = g(x)$ and $g'(x) = -f(x)$, then $h'(x) =$

(A) $f(x)g(x)$
(B) $2f(x) - f(x)g(x)$
(C) $[f(x) + g(x)]^2$
(D) $[f(x) - g(x)]^2$
(E) $[g(x)]^2 + 2g(x)f(x) - [f(x)]^2$

Ans

16. Which of the following integrals gives the length of the graph of $y = \arcsin\frac{x}{2}$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$?

(A) $\int_a^b \sqrt{1 - \frac{1}{\sqrt{4 - x^2}}}\, dx$

(B) $\int_a^b \sqrt{1 + \frac{1}{\sqrt{4 - x^2}}}\, dx$

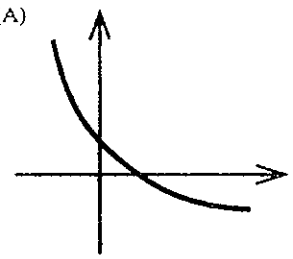
(C) $\int_a^b \sqrt{1 - \frac{1}{4 - x^2}}\, dx$

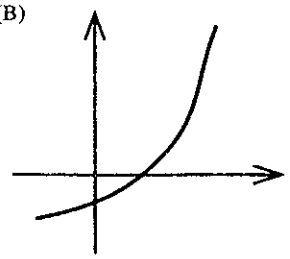
(D) $\int_a^b \sqrt{1 + \frac{1}{4 - x^2}}\, dx$

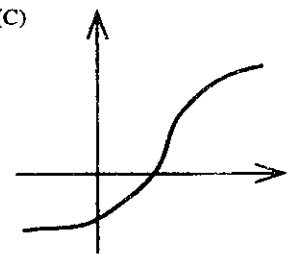
(E) $\int_a^b \left[1 + \frac{1}{4 - x^2}\right] dx$

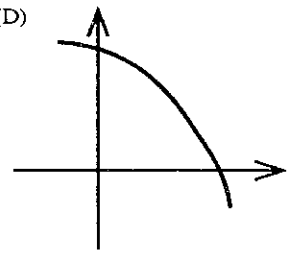
Ans

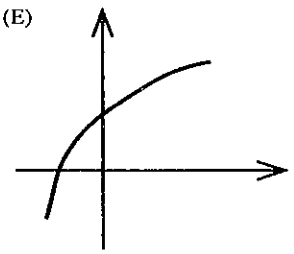
17. If y is a function of x such that $\frac{dy}{dx} > 0$ for all x and $\frac{d^2y}{dx^2} < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?

(A) 

(B) 

(C) 

(D) 

(E) 

Ans

18. $\int \frac{1}{x^2 + x}\, dx =$

(A) $\frac{1}{2} \arctan\left(x + \frac{1}{2}\right) + C$

(B) $\ln\left|x^2 + x\right| + C$

(C) $\ln\left|\frac{x + 1}{x}\right| + C$

(D) $\ln\left|\frac{x}{x + 1}\right| + C$

(E) none of these

Ans

19. If $f(x) = x^2e^{-2x}$, then the graph of f is increasing for all x such that

(A) $0 < x < 1$ (B) $0 < x < \frac{1}{2}$ (C) $0 < x < 2$ (D) $x < 0$ (E) $x > 0$

Ans

20. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ is the Taylor series about $x = 0$ for which of the following functions?

(A) $\sin x$

(B) $\cos x$

(C) e^x

(D) e^{-x}

(E) $\ln(1 + x)$

Ans

21. Evaluate $\int_{-\pi/4}^{e-1} f(x)\,dx$ if $f(x) = \begin{cases} \sec^2 x & \text{for } x \le 0 \\ \frac{1}{x+1} & \text{for } x > 0 \end{cases}$

(A) 0

(B) 1

(C) 2

(D) e

(E) π

Ans

22. $\lim_{x \to 0} \left(1 + \frac{x}{2}\right)^{\cot x} =$

(A) $\frac{e}{2}$

(B) e

(C) $\sqrt{e}$

(D) $2e$

(E) e^2

Ans

23. The area of the closed region bounded by the polar graph of $r = \sqrt{1+\cos\theta}$ is given by

(A) $\int_0^{2\pi} \sqrt{1+\cos\theta}\ d\theta$

(B) $\int_0^{\pi} \sqrt{1+\cos\theta}\ d\theta$

(C) $2\int_0^{2\pi} (1+\cos\theta)\ d\theta$

(D) $\int_0^{\pi} (1+\cos\theta)\ d\theta$

(E) $2\int_0^{\pi} \sqrt{1+\cos\theta}\ d\theta$

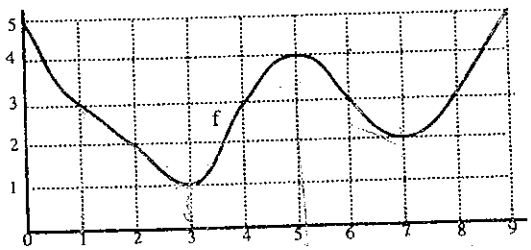
Ans

24. $\lim_{h\to 0}\left[\dfrac{(3+h)^5 - 3^5}{9h}\right]$ is

(A) 0
(B) 1
(C) 45
(D) 405
(E) nonexistent

Ans

25. Consider the function f whose graph is shown at the right. Use the Trapezoid Rule with $n = 4$ to estimate the value of $\int_1^9 f(x)\,dx$.



(A) 21
(B) 22
(C) 23
(D) 24
(E) 25

Ans

26. $\int x\sqrt{1 - x^2}\,dx =$

(A) $\dfrac{(1-x^2)^{3/2}}{3} + C$ (B) $-(1-x^2)^{3/2} + C$ (C) $\dfrac{x^2(1-x^2)^{3/2}}{3} + C$

(D) $-\dfrac{x^2(1-x^2)^{3/2}}{3} + C$ (E) $-\dfrac{(1-x^2)^{3/2}}{3} + C$

Ans

27. Suppose a continuous function f and its derivative f' have values as given in the following table. Given that $f(1) = 2$, use Euler's method to approximate the value of $f(2)$.

x	1.0	1.5	2.0
$f'(x)$	0.4	0.6	0.8
$f(x)$	2.0		

(A) 2.1 (B) 2.3 (C) 2.5 (D) 2.7 (E) 2.9

Ans

28. A particle is moving in the xy-plane and its position at time t is given by $x = \cos\left(\frac{\pi}{3}t\right)$ and $y = 2\sin\left(\frac{\pi}{3}t\right)$. When $t = 3$, the speed of the particle is

(A) $-\frac{2\pi}{3}$ (B) 0 (C) $\frac{2\pi}{3}$ (D) π (E) $\frac{\sqrt{13}}{3}\pi$

Ans