EXAM I
CALCULUS BC
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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1. The slope of the curve $y = x^2 - e^{-x}$ at its point of inflection is

(A) $-\ln 2$ (B) $-\ln 4$ (C) $2 - \ln 4$ (D) $2 + \ln 4$ (E) $\frac{e^2}{2}$

Ans ☐

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2. If the graph of the parabola $y = 2x^2 + x + k$ is tangent to the line $3x + y = 1$, then $k =$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Ans ☐

3. If a function f is defined by $f(x) = \int_0^x \frac{1}{1+t^4}\,dt$, which of the following statements are true?

I. $f(1) = \frac{1}{2}$

II. the graph of f is concave down at $x = 3$.

III. $f(x) + f(-x) = 0$ for all real numbers x.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II and III

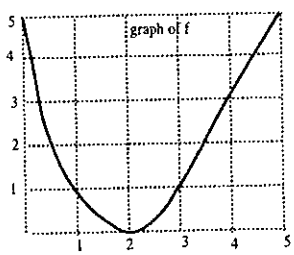
Ans

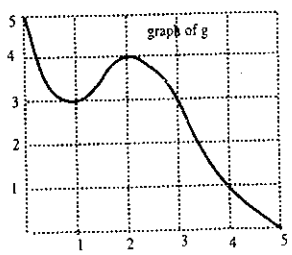
4. Consider the function f defined on the domain $-0.5 \le x \le 0.5$ with $f(0) = 1$, and $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \sec^2(3x)$. Evaluate: $\int_0^{0.5} f(x)\,dx$.

(A) 0.294
(B) 0.794
(C) 1.294
(D) 1.794
(E) 4.700

Ans

5. The graphs of functions f and g are shown at the right. The graph of g has a horizontal tangent at $x = 1$.
If $h(x) = f[g(x)]$, which of the following statements are true about the function h?





I. $h(2) = 5$.

II. h is increasing at $x = 4$.

III. The graph of h has a horizontal tangent at $x = 1$.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II and III

Ans

6. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = \cos(t + \sqrt{t})$. The total distance traveled by the particle from $t = 0$ to $t = 4$ is

(A) 0.481 (B) 1.069 (C) 1.449 (D) 1.932 (E) 2.416

Ans

7. The total area of the region bounded by the graphs of $y = \arctan x$ and $y = 4 - x^2$ is

(A) 10.802 (B) 10.972 (C) 11.142 (D) 11.31 (E) 11.482

Ans

8. Which of the following series are conditionally convergent?

I. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n + 1}$ II. $\sum_{n=1}^{\infty} (-1)^n \frac{\cos n}{3^n}$ III. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, III

Ans

9. If $x = t^2$ and $y = \ln(t^2 + 1)$, then at $t = 1$, $\dfrac{d^2y}{dx^2}$ is

(A) $-\dfrac{1}{4}$ (B) $-\dfrac{1}{2}$ (C) -1 (D) 0 (E) $\dfrac{1}{4}$

Ans

10. When a wholesale produce market has x crates of lettuce available on a given day, it charges p dollars per crate as determined by the supply equation $px - 20p - 6x + 40 = 0$. If the daily supply is decreasing at the rate of 8 crates per day, at what rate is the price changing when the supply is 100 crates?

(A) not changing
(B) increasing at \$0.10 per day
(C) decreasing at \$0.10 per day
(D) increasing at \$1.00 per day
(E) decreasing at \$1.00 per day

Ans

11. Let R be the region in the first quadrant bounded above by the graph of $f(x) = 2\text{Arc}\tan x$ and below by the graph of $y = x$. What is the volume of the solid generated when R is rotated about the x-axis?

(A) 1.217 (B) 2.276 (C) 2.693 (D) 6.666 (E) 7.151

Ans

12. For any time $t \geq 0$, if the position of a particle in the xy-plane is given by $x = e^t$ and $y = e^{-t}$, then the speed of the particle at time $t = 1$ is

(A) 2.693 (B) 2.743 (C) 3.086 (D) 3.844 (E) 7.542

Ans

13. The level of air pollution at a distance x miles from a tire factory is given by

$$L(x) = e^{-0.1x} + \frac{1}{x^2}.$$

The average level of pollution between 15 and 25 miles from the factory is

(A) 0.144
(B) 0.156
(C) 0.162
(D) 0.168
(E) 0.250

Ans

14. What is the x-coordinate of the point on the curve $y = e^x$ that is closest to the origin?

(A) -0.452 (B) -0.426 (C) -0.400 (D) -0.374 (E) -0.372

Ans

15. Let $f(x) = e^{x/2}$. If the second-degree Taylor polynomial for f about $x = 0$ is used to approximate f on the interval $[0,2]$, what is the Lagrange error bound for the maximum error on the interval $[0,2]$?

(A) 0.028 (B) 0.113 (C) 0.453 (D) 0.499 (E) 0.517

Ans

16. If $\int f(x) \cdot \cos x \, dx = f(x) \cdot \sin x - \int 6x^2 \sin x \, dx$, then $f(x)$ could be

(A) $-2x^3$

(B) $2x^3$

(C) $-3x^2$

(D) $3x^2$

(E) $x \sin x$

Ans

17. Which of the following statements are true?

I. If the graph of a function is always concave up, then the left-hand Riemann sums with the same subdivisions over the same interval are always less than the right-hand sums.

II. If the function f is continuous on the interval $[a, b]$ and $\int_a^b f(x) \, dx = 0$, then f must have at least one zero between a and b.

III. $f'(x) > 0$ for all x in an interval, then the function f is concave up in that interval.

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) none

Ans