**EXAM II**
**CALCULUS BC**
**SECTION I PART A**
**Time–55 minutes**
**Number of questions–28**

**A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION**

**Directions:** Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

**In this test:**

(1) Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function $f$ may be indicated using the inverse function notation $f^{-1}$ or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

---

1. $\int_0^1 xe^{x^2}\,dx$

(A) $\frac{1}{2}(e-1)$ (B) $2(e-1)$ (C) $2e$ (D) $e$ (E) $\frac{1}{2}e$

Ans ☐

---

2. If $f(x) = x^2 - 1$, then $\lim_{x \to 1} \frac{f(x+1) - f(2)}{x^2 - 1}$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) nonexistent

Ans ☐

3. If $\cos x = e^y$ and $0 < x < \pi$, then $\frac{dy}{dx}$ is

(A) $-\tan x$

(B) $-\cot x$

(C) $\tan x$

(D) $\cot x$

(E) $\csc x$

Ans

4. If $y = \text{Arcsin}(e^{2x})$, then $\frac{dy}{dx} =$

(A) $\frac{2e^{2x}}{\sqrt{1 - e^{4x}}}$

(B) $\frac{e^{2x}}{\sqrt{1 - e^{4x}}}$

(C) $\frac{2e^{2x}}{\sqrt{1 + e^{4x}}}$

(D) $\frac{e^{2x}}{1 - e^{4x}}$

(E) $\frac{2e^{2x}}{\sqrt{e^{4x} - 1}}$

Ans

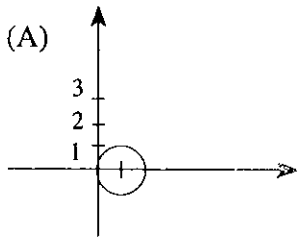
5. If $f(x) = \int_2^{2x} \frac{1}{\sqrt{t^3 + 1}}\, dt$, then $f'(1) =$

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\sqrt{2}$ (E) undefined

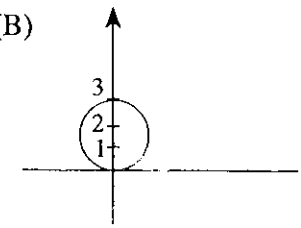
Ans

6. Which of the following represents the graph of the polar curve $r = 3\csc\theta$?

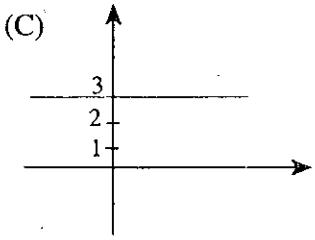
(A)



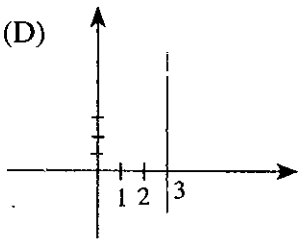

(B)



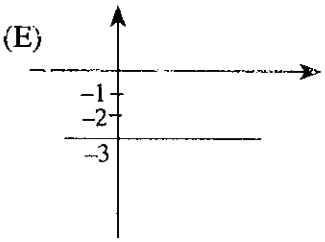

(C)




(D)




(E)




Ans

7. If $g(x) = \tan^2(e^x)$, then $g'(x)=$

(A) $2e^x \tan(e^x)\sec^2(e^x)$

(B) $2\tan(e^x)\sec^2(e^x)$

(C) $2\tan^2(e^x)\sec(e^x)$

(D) $e^x \sec^2(e^x)$

(E) $2e^x \tan(e^x)$

Ans

8. $\int_0^1 \sqrt{x^2 - 2x + 1}\, dx =$

(A) $-1$ (B) $-\frac{1}{2}$ (C) $0$ (D) $\frac{1}{2}$ (E) $1$

Ans

9. The coefficient of $x^3$ in the Taylor series for $f(x)=\int_0^x \frac{1+t}{e^t}\,dt$ about $x=0$ is

(A) $-1$ (B) $-\frac{1}{6}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) 1

Ans

10. $F$ and $G$ are two functions whose derivatives exist for all real $x$; $F'(x)<0$ and $G'(x)>0$ for all real $x$. Which of the following could be true about the graphs of $y=F(x)$ and $y=G(x)$?

I. they do not intersect II. they intersect once III. they intersect more than once

(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

Ans

11. The length of the curve determined by the parametric equations $x=\sin t$ and $y=t$ from $t=0$ to $t=\pi$ is

(A) $\int_0^{\pi} \sqrt{\cos^2 t+1}\,dt$

(B) $\int_0^{\pi} \sqrt{\sin^2 t+1}\,dt$

(C) $\int_0^{\pi} \sqrt{\cos t+1}\,dt$

(D) $\int_0^{\pi} \sqrt{\sin t+1}\,dt$

(E) $\int_0^{\pi} \sqrt{1-\cos t}\,dt$

Ans

12. $\lim_{x \to 0} \dfrac{e^{3x} - 1}{\sin x} =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) nonexistent

Ans

13. The slope of the line tangent to the graph of $\ln(x + y) = x^2$ at the point where $x = 1$ is

(A) 0 (B) 1 (C) e – 1 (D) 2e – 1 (E) e – 2

Ans

14. At $x = 0$, which of the following is true of the function $f(x) = \sin x + e^{-x}$ ?

(A) $f$ is increasing
(B) $f$ is decreasing
(C) $f$ is discontinuous
(D) The graph of $f$ is concave up
(E) The graph of $f$ is concave down

Ans

15. The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{n+1}{2n+1} \cdot \frac{(x-3)^n}{2^n}$ is

(A) 4
(B) 3
(C) 2
(D) 1
(E) 0

Ans

16. A particle moves along the curve $x^2y = 2$ at time $t > 0$. If $\frac{dy}{dt} = 8$ when $x = -1$, what is the value of $\frac{dx}{dt}$ at that time?

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

Ans

17. If $\int_a^b f(x)\,dx = 3$ and $\int_a^b g(x)\,dx = 2$, which of the following must be true?

I. $f(x) > g(x)$ for all $a \le x \le b$

II. $\int_a^b [f(x) - g(x)]\,dx = 1$

III. $\int_a^b [f(x) \cdot g(x)]\,dx = 6$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, III

Ans

18. Consider the curve in the $xy$-plane represented by $x = \frac{2}{t}$ and $y = \ln t$ for $t > 0$. The slope of the line tangent to the curve at the point where $x = 1$ is

(A) $-1$ (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

Ans

19. If $\frac{dy}{dx} = xy^2$, then $y$ could be

(A) $\frac{-1}{x^2 + 1}$

(B) $-\frac{1}{x^2} + 1$

(C) $\frac{-2}{x^2 + 1}$

(D) $3e^{x^2/2}$

(E) $3e^{x^2/2} + 1$

Ans

20. $\int \frac{x}{x + 2}\, dx$

(A) $x \ln|x+2| + C$

(B) $x+2 \ln|x+2| + C$

(C) $x-2 \ln|x+2| + C$

(D) $x- \ln|x+2| + C$

(E) $x- \text{Arctan } x + C$

Ans

21. Let $f$ be a function with $f(2) = 4$ and derivative $f'(x) = \sqrt{x^3 + 1}$. Using a tangent line approximation to the graph of $f$ at $x = 2$, estimate $f(2.2)$.

(A) 4.0 (B) 4.2 (C) 4.4 (D) 4.6 (E) 4.8

Ans

22. A region in the plane is bounded by $y = \frac{1}{\sqrt{x}}$, the $x$-axis, the line $x = m$ and the line $x = 2m$ where $m > 0$. A solid is formed by revolving the region about the $x$-axis. The volume of this solid

(A) is independent of $m$

(B) increases as $m$ increases

(C) decreases as $m$ increases

(D) increases until $m = \frac{1}{2}$, then decreases

(E) is none of the above

Ans

23. If a particle moves in the $xy$-plane so that at time $t > 0$ its position vector is $\left\langle \sin(3t - \frac{\pi}{2}),\ 3t^2 \right\rangle$, then at time $t = \frac{\pi}{2}$ the velocity vector is

(A) $\langle -3, 3\pi \rangle$ (B) $\langle -1, 3\pi \rangle$ (C) $\langle -1, 2\pi \rangle$ (D) $\langle 3, 2\pi \rangle$ (E) $\langle 3, 3\pi \rangle$

Ans

24. The value of the derivative of $y = \dfrac{(x^2 - 3)^3}{(5x - 9)^2}$ at $x = 2$ is

(A) $-4$ (B) $-2$ (C) 0 (D) 2 (E) 4

Ans

25. $F$ and $G$ are differentiable functions such that $F(x) = \int_0^x G(t)\,dt$. If $F(a) = 3$ and $F(b) = 3$, where $0 < a < b$, which of the following must be true?

(A) $G(x) = 0$ for some $x$ such that $a < x < b$

(B) $G(x) = 0$ for all $x$ such that $a < x < b$

(C) $G(x) > 0$ for all $x$ such that a < x < b

(D) $F(x) \geq 0$ for all x such that $a < x < b$

(E) $F(x) = 0$ for some $x$ such that $a < x < b$

Ans

26. $\int_0^1 xe^{-x}\,dx =$

(A) 1 (B) $1-\frac{2}{e}$ (C) $\frac{2}{e}-1$ (D) $1+\frac{2}{e}$ (E) $-\frac{2}{e}$

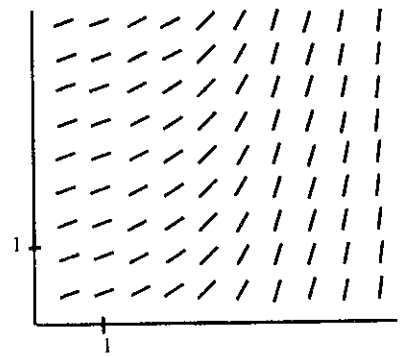
Ans

27. If the average rate of change of a function $f$ over the interval from $x=2$ to $x=2+h$ is given by $7e^h - 4\cos(2h)$, then $f'(2) =$

(A) $-1$ (B) 0 (C) 1 (D) 2 (E) 3

Ans

28. The slope field shown in the figure at the right represents solutions to a certain differential equation. Which of the following could be a specific solution to that differential equation ?




(A) $y = e^{-x}$

(B) $y = \sin x$

(C) $y = \sqrt{x}$

(D) $y = \ln x$

(E) $y = e^{0.5x}$

Ans