EXAM III
CALCULUS BC
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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1. A particle moves on the x-axis in such a way that its position at time t, $t > 0$, is given by $x(t) = (\ln x)^2$. At what value of t does the velocity of the particle attain its maximum?

(A) 1 (B) $e^{1/2}$ (C) e (D) $e^{3/2}$ (E) e^2

Ans ☐

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2. Which of the following is equal to $\int_0^{\pi} \cos x\,dx$?

(A) $\int_0^{\pi} \sin x\,dx$ (B) $\int_{-\pi/2}^{\pi/2} \cos x\,dx$ (C) $\int_{-\pi/2}^{\pi/2} \sin x\,dx$

(D) $\int_{\pi}^{2\pi} \sin x\,dx$ (E) $\int_{\pi/2}^{3\pi/2} \cos x\,dx$

Ans ☐

3. A solution to $\frac{dy}{dx} = \frac{1}{xy}$ that goes through the point $(1, 1)$ is

(A) $\frac{1}{x^2}$

(B) $\sqrt{2\ln x} + 1$

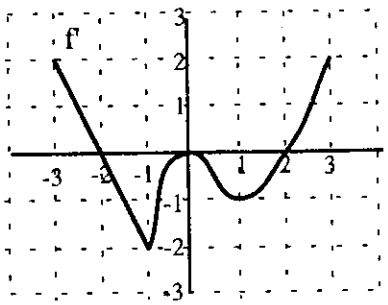
(C) $\sqrt{2\ln x + 1}$

(D) $\sqrt{\ln x + 1}$

(E) e^{x-1}

Ans

4. At the right is the graph of $y = f'(x)$, the derivative of $y = f(x)$. The domain of f is the interval $-3 \le x \le 3$. Which of the following must be true about the graph of f?



I. f is increasing on $-3 < x < -2$.

II. The graph of f is concave down on $-3 < x < -1$.

III. The maximum value of $f(x)$ on the interval $-3 < x < 2$ is $f(-3)$.

(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

Ans

5. $\int_e^{+\infty} \frac{1}{x(\ln x)^2}\, dx$

(A) $\frac{1}{e}$ (B) $-\frac{1}{e}$ (C) e (D) 1 (E) divergent

Ans

6. If $x = t^2$ and $y = (t^2 + 1)^2$, then at $t = 3$, $\frac{dy}{dx}$ is

(A) 0

(B) $\frac{5}{3}$

(C) 6

(D) 20

(E) undefined

Ans

7. Suppose a population of bears grows according to the logistic differential equation

$$\frac{dP}{dt} = 2P - 0.01P^2$$

where P is the number of bears at time t in years. Which of the following statements are true?

I. The growth rate of the bear population is greatest at $P = 100$.

II. If $P > 200$, the population of bears is decreasing.

III. $\lim_{t \to \infty} P(t) = 200$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Ans

8. The substitution of $x = \sin\theta$ in the integral $\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}}\,dx$, results in

(A) $\int_0^{1/2} \frac{\sin^2\theta}{\cos\theta}\,d\theta$ (B) $\int_0^{1/2} \sin^2\theta\,d\theta$ (C) $\int_0^{\pi/6} \sin^2\theta\,d\theta$

(D) $\int_0^{\pi/3} \sin^2\theta\,d\theta$ (E) $\int_0^{1/2} \frac{\cos^2\theta}{\sin\theta}\,d\theta$

Ans

9. Let f and g be functions whose derivatives exist for all real numbers, with $g(x) \neq 0$ for $x \neq 0$.

If $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} g(x) = 0$ and $\lim_{x\to 0} f'(x) = 6$ and $\lim_{x\to 0} g'(x) = 2$, then $\lim_{x\to 0} \frac{f(x)}{g(x)}$ is

(A) 0

(B) 1

(C) 3

(D) $\frac{f'(x)}{g'(x)}$

(E) nonexistent

Ans

10. The slope of the tangent to the graph of $y = \text{Arc}\tan\frac{x}{2}$ at $(2, \frac{\pi}{4})$ is

(A) $\frac{1}{16}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 1

Ans

11. If $\frac{dy}{dx} = \frac{3 \sin x}{\sec^2 x}$, then $y =$

(A) $\ln|\cos x| + C$

(B) $\sec x + C$

(C) $\cos^3 x + C$

(D) $-3\cos^3 x + C$

(E) $-\cos^3 x + C$

Ans

12. Consider the set of all right circular cylinders for which the sum of the height and the diameter is 18 inches. What is the radius of the cylinder with the maximum volume?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Ans

13. The total area of the region enclosed by the polar graph of $r = 1 + \sin\theta$ is

(A) $\frac{\pi}{2}$

(B) π

(C) $\frac{3\pi}{2}$

(D) 2π

(E) $\frac{5\pi}{2}$

Ans

14. The acceleration of a particle moving along the x-axis at any time $t \geq 0$ is given by $a(t) = 1 + e^{-t}$. At $t = 0$ the velocity of the particle is -2 and its position is 3. The position of the particle at any time t is

(A) $\frac{t^2}{2} - t + e^{-t} + 2$

(B) $\frac{t^2}{2} - 3t + e^{-t} + 2$

(C) $\frac{t^2}{2} - t - e^{-t} + 2$

(D) $\frac{t^2}{2} - 3t - e^{-t} + 2$

(E) $t^2 - t + e^{-t} + 2$

Ans

15. Which of the following integrals gives the length of the graph of $y = \tan x$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$?

(A) $\int_a^b \sqrt{x^2 + \tan^2 x}\, dx$

(B) $\int_a^b \sqrt{x + \tan x}\, dx$

(C) $\int_a^b \sqrt{1 + \sec^2 x}\, dx$

(D) $\int_a^b \sqrt{1 + \tan^2 x}\, dx$

(E) $\int_a^b \sqrt{1 + \sec^4 x}\, dx$

Ans

16. If $v = \sin(u^2 - 1)$ and $u = \sqrt{x^2 + 1}$, then $\frac{dv}{dx}$ is

(A) $\frac{\cos(x^2)}{2\sqrt{x^2+1}}$

(B) $\frac{x\cos(x^2)}{2\sqrt{x^2+1}}$

(C) $\frac{x\cos(x^2-1)}{\sqrt{x^2+1}}$

(D) $2x\cos(x^2)$

(E) $\cos(x^2)$

Ans

17. The function f is continuous at the point $(c, f(c))$. Which of the following statements could be false?

(A) $\lim_{x\to c} f(x)$ exists (B) $\lim_{x\to c} f(x) = f(c)$ (C) $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$

(D) $f(c)$ is defined (E) $f'(c)$ exists

Ans

18. The area of the region in the first quadrant under the curve $y = \frac{1}{\sqrt{1-x^2}}$ bounded on the left by $x = \frac{1}{2}$ and on the right by $x = 1$ is

(A) ∞

(B) π

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{3}$

(E) none of these

Ans

19. The function f is defined by $f(x) = 3x^2 - x^3 + h$. For which values of h will f have three distinct zeros ?

(A) all $h > 4$
(B) $0 < h < 4$
(C) all $h < 0$
(D) $-4 < h < 0$
(E) all $h < -4$

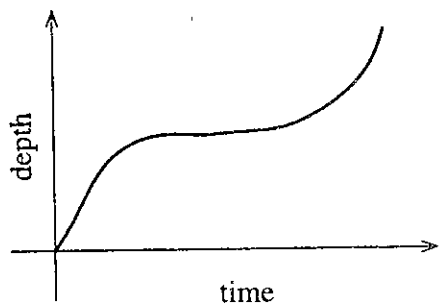
Ans

20. An isosceles triangle has one vertex at the origin and the other two at the points where a line parallel to and above the x-axis intersects the curve $y = 12 - x^2$. The maximum area of the triangle is

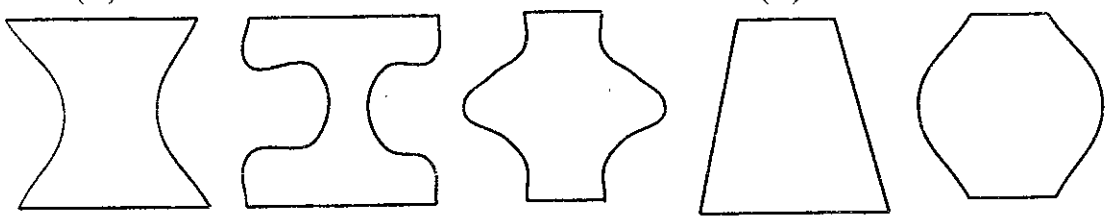
(A) 40 (B) 32 (C) 24 (D) 16 (E) 8

Ans

21. Every cross section perpendicular to the axis of a container is a circle. Water is flowing into the container at a constant rate. A graph of the depth of the water as a function of time is shown at the right. Which of the following best describes the profile of the container?



(A) (B) (C) (D) (E)



Ans

22. The sales of a small company are expected to grow at a rate given by $\frac{dS}{dt} = 300t + t^{1/2} + t^{3/2}$, where $S(t)$ is the sales in dollars in t days. The accumulated sales from the first day through the fourth day is approximately

(A) \$2400
(B) \$2406
(C) \$2412
(D) \$2418
(E) \$2424

Ans

23. If $F(x) = \int_{\pi/2}^{x} 4t \sin\left(\frac{t}{3}\right) dt$, then an equation of the line tangent to $y = F(x)$ at the point where $x = \frac{\pi}{2}$ is

(A) $2x - \pi y - \pi = 0$
(B) $2x - 2y - \pi = 0$
(C) $2\pi x - 2y - \pi^2 = 0$
(D) $\pi x - 2y - \pi^2 = 0$
(E) $\pi x - y - \pi = 0$

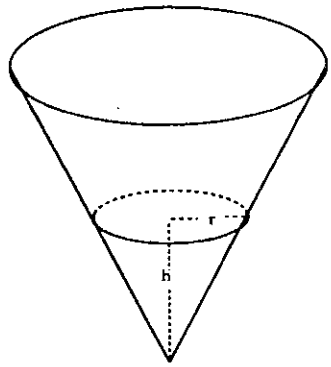
Ans

24. If $\int_{0}^{k} \frac{\sec^2 x}{1 + \tan x}\, dx = \ln 2$, then the value of k is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) π

Ans

25. A conical tank is being filled with water at the rate of 16 ft^3/min. The rate of change of the depth of the water is 4 times the rate of change of the radius of the water surface. At the moment when the depth is 8 ft and the radius of the surface is 2 ft, the area of the surface is changing at the rate of



Volume of Cone = $\frac{1}{3}\pi r^2 h$

(A) $\frac{1}{\pi}$ ft^2/min

(B) 1 ft^2/min

(C) 4 ft^2/min

(D) 4π ft^2/min

(E) 16π ft^2/min

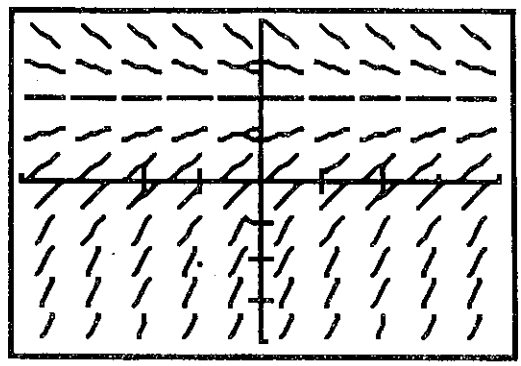
Ans

26. Given the differential equation $\frac{dy}{dx} = \frac{1}{x+1}$ and $y(0) = 0$. An approximation of $y(1)$ using Euler's method with two steps and step size $\Delta x = 0.5$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$ (E) $\frac{9}{10}$

Ans

27. A slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given at the right. Which of the following could be a solution ?



(A) $y = 2 + \ln x$

(B) $y = 2 - \ln x$

(C) $y = 2 - e^{x}$

(D) $y = 2 - e^{-x}$

(E) $y = 2 + e^{2x}$

Ans

28. $\int x\, e^{2x}\, dx =$

(A) $\frac{1}{4}\, e^{2x}(2x-1) + C$

(B) $\frac{1}{2} e^{2x}(2x-1) + C$

(C) $\frac{1}{4} e^{2x}(4x-1) + C$

(D) $\frac{1}{2} e^{2x}(x-1) + C$

(E) $\frac{1}{4}\, e^{2x}(x-1) + C$

Ans