EXAM III
CALCULUS BC
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

---

1. Which of the following must be true about a particle that starts at $t = 0$ and moves along a number line if its position at time t is given by $s(t) = (t-2)^3(t-6)$?

I. The particle is moving to the right for $t > 5$.
II. The particle is at rest at $t = 2$ and $t = 6$.
III. The particle changes direction at $t = 2$.

(A) I only (B) II only (C) III only (D) I and III only (E) none

Ans ☐

---

2. The approximate *average* rate of change of the function $f(x) = \int_0^x \sin(t^2)\,dt$ over the interval $[1, 3]$ is

(A) 0.155 (B) 0.232 (C) 0.309 (D) 0.386 (E) 0.463

Ans ☐

---

3. $\int \frac{1}{\sqrt{x}\,(1-\sqrt{x})}\,dx =$

(A) $\frac{1}{2}\ln\left|1-\sqrt{x}\right|+C$

(B) $2\ln\left|1-\sqrt{x}\right|+C$

(C) $4\sqrt{1-\sqrt{x}}\,+C$

(D) $-2\ln\left|1-\sqrt{x}\right|+C$

(E) none of these

Ans

4. Let R be the region in the first quadrant that is enclosed by the graph of $f(x)=\ln(x+1)$, the x-axis and the line $x=e$. What is the volume of the solid generated when R is rotated about the line $y=-1$?

(A) 5.037 (B) 6.545 (C) 10.073 (D) 20.146 (E) 28.686

Ans

5. $\lim_{h\to 0} \frac{\int_1^{1+h}\sqrt{x^3+8}\,dx}{h}$ is

(A) 0

(B) 1

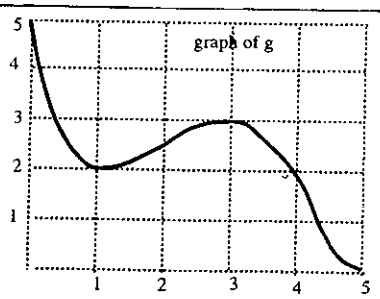
(C) 3

(D) $2\sqrt{2}$

(E) nonexistent

Ans

6. A graph of the function g is shown in the figure. If the function h is defined by $h(x) = g(x^2)$, use the graph to estimate $h'(2)$.



(A) –8 (B) –4 (C) –2 (D) 2 (E) 4

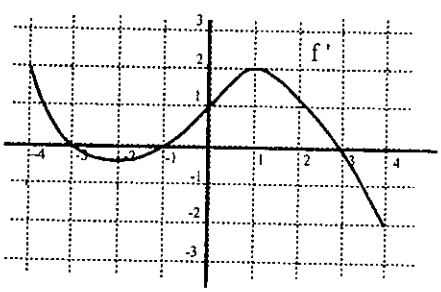
Ans

7. Let f be a function having 5 derivatives on the interval [2, 2.9] and assume that $|f^{(5)}(x)| \le 0.8$ for all x in the interval [2, 2.9]. If the fourth-degree Taylor polynomial for f about $x = 2$ is used to approximate f on the interval [2, 2.9], what is the Lagrange error bound for the maximum error on the interval [2, 2.9]?

(A) 0.004 (B) 0.011 (C) 0.022 (D) 0.033 (E) 0.044

Ans

8. A function f is defined on the closed interval $-4 \le x \le 4$. The graph of f', the derivative of f, is shown at the right. If the graph of f' has horizontal tangents at $x = -2$ and $x = 1$, which of the following must be true about the original function f?



The derivative of f

I. f is increasing on the interval (–2, 1).

II. f is continuous at $x = 0$.

III. The graph of f has an inflection point at $x = -2$.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II and III

Ans

9. A curve is defined parametrically by $x = e^t$ and $y = 2e^{-t}$. An equation of the tangent line to the curve at $t = \ln 2$ is

(A) $x - 2y + 3 = 0$

(B) $x + 2y - 4 = 0$

(C) $x + 2y - 5 = 0$

(D) $x - 2y - 4 = 0$

(E) $2x + y - 5 = 0$

Ans

10. If $x^2 - y^2 = 25$ then $\frac{d^2y}{dx^2} =$

(A) $-\frac{x}{y}$ (B) $\frac{5}{y^2}$ (C) $-\frac{x^2}{y^3}$ (D) $-\frac{25}{y^3}$ (E) $\frac{4}{y^3}$

Ans

11. Which of the following series are convergent?

I. $1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots + \frac{1}{n^2} + \ldots$

II. $1 - \frac{1}{2} + \frac{1}{3} - \ldots + \frac{(-1)^n}{n} + \ldots$

III. $2 + 1 + \frac{8}{9} + \ldots + \frac{2^n}{n^2} + \ldots$

(A) I only

(B) III only

(C) I and II only

(D) II and III only

(E) I, II and III

Ans

12. If $\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \frac{x^2+1}{x^2}$, then $g(x)$ could be equal to

(A) x^{-3} (B) $-2x^{-3}$ (C) $\frac{x^2-1}{x}$ (D) $x - x^2$ (E) $1 + x^{-2}$

Ans

13. Two particles move along the x-axis and their positions at time $0 \le t \le 2\pi$ are given by $x_1 = \cos t$ and $x_2 = e^{(t-3)/2} - 0.75$. For how many values of t do the two particles have the same velocity?

(A) 0

(B) 1

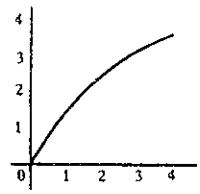
(C) 2

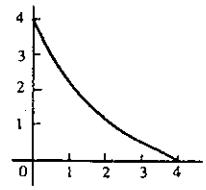
(D) 3

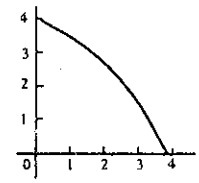
(E) 4

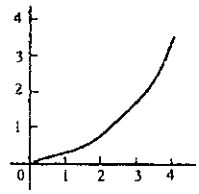
Ans

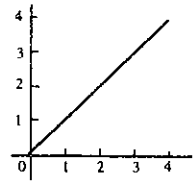
14. If a left Riemann sum overapproximates the definite integral $\int_0^4 f(x)\,dx$ and a trapezoid sum underapproximates the integral $\int_0^4 f(x)\,dx$, which of the following could be a graph of $y = f(x)$?

(A) 

(B) 

(C) 

(D) 

(E) 

Ans

15. The radius of convergence of the series $x + \frac{2x^2}{2^2} + \frac{6x^3}{3^3} + \cdots + \frac{n!x^n}{n^n} + \cdots$ is

(A) ∞ (B) e^2 (C) e (D) $\frac{e}{2}$ (E) 0

Ans

16. When using the method of partial fractions to decompose $\frac{8x-4}{x^2+2x-3}$, one of the fractions obtained is

(A) $\frac{1}{x+3}$ (B) $\frac{7}{x-1}$ (C) $\frac{7}{x+3}$ (D) $\frac{1}{x-3}$ (E) $\frac{7}{x+1}$

Ans

17. A particle moves on the xy-plane so that at time $t \geq 0$ its acceleration vector is $\langle 2, e^{-t} \rangle$. When $t = 0$, the particle is at rest and its position is $\langle 3,3 \rangle$. At $t = 2$ the position of the particle is

(A) $\langle 4, e^{-2} \rangle$ (B) $\langle 4, 2+e^{-2} \rangle$ (C) $\langle 7, e^{-2} \rangle$ (D) $\langle 7, 2+e^{-2} \rangle$ (E) $\langle 7, 4+e^{-2} \rangle$

Ans