Multiple Choice Solutions

① $y = \frac{x-3}{2-5x}$

$\frac{dy}{dx} = \frac{(2-5x)(1)-(x-3)(-5)}{(2-5x)^2} = \frac{2-5x+5x-15}{(2-5x)^2} = \boxed{\frac{-13}{(2-5x)^2}}$ E

② $y = 2\sqrt{x} - \frac{1}{2\sqrt{x}}$

$y = 2x^{1/2} - \frac{1}{2}x^{-1/2}$

$y' = x^{-1/2} + \frac{1}{4}x^{-3/2}$

$\boxed{y' = \frac{1}{\sqrt{x}} + \frac{1}{4x\sqrt{x}}}$ D

③ $f(x) = |x^2 - 4|$

not diffeentiable at

sharp point at $\boxed{x = \pm 2}$ C

④ All choices could be true, E

but do not have to be true

⑤ $y = \sqrt{3-2x}$

$y = (3-2x)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(3-2x)^{-1/2}(-2)$

$\boxed{\frac{dy}{dx} = \frac{-1}{\sqrt{3-2x}}}$ D

⑥ $f(x) = m - 2kx$

$f'(x) = -2k$

$f'(j) = \boxed{-2k}$ D

plug j in for "x", but there is no x.

⑦ $y = 6x^{1/2}$ (0,0)

$y' = 3x^{-1/2}$

$y'(0) = \frac{3}{\sqrt{0}} = \boxed{\text{undefined}}$ E

⑧ $g(x)=\ln x \quad g'(x)=?$

This problem uses the def. of derivative

$$f'(x)=\lim_{k\to 0}\frac{f(x+k)-f(x)}{k}$$

$$f'(x)=\boxed{\lim_{k\to 0}\frac{\ln(x+k)-\ln x}{k}} \quad D$$

⑨ $f(x)=x^3$ use chain rule $f'(x)=3x^2$

$$\frac{d}{dx}[f(f(f(g(x))))]$$

$$\frac{d}{dx}f(g(x))=f'(g(x))(g'(x))$$

$$=3(g(x))^2(g'(x))$$

$$f(f(g(x)))=[3(g(x))^2 g'(x)]^3 \quad E$$

⑩ $xy=20 \quad y'(t)=10 \quad x=2$ find $x'(t)$

$x(y')+y(x')=0$ product rule

$2(10)+10x'=0$ if $x=2$ $y=10$

$20+10x'=0$

$\boxed{x'=-2}$ A

⑪ $f(x)=\tan 5x \quad f'(\pi/5)$

$f'(x)=\sec^2(5x)(5)$

$f'(\frac{\pi}{5})=5\sec^2(\pi)$

$=5(1)=\boxed{5}$ A

(12) $y = \sin^3(1-2x)$
$y = [\sin(1-2x)]^3$
$\frac{dy}{dx} = 3[\sin(1-2x)]^2 (\cos(1-2x))(-2)$
$\frac{dy}{dx} = -6\,(\sin^2(1-2x))(\cos(1-2x))$ D

(13) $f(x) = \sin x \quad g(x) = \cos x$
$f'(x) = \cos x \quad g'(x) = -\sin x$
$\cos x = -\sin x$
$x = \boxed{\frac{3\pi}{4} + k\pi}$ (quadrants II and IV) C

(14) $y = 3x^{50}$
$y' = 3 \cdot 50\, x^{49}$
$y'' = 3 \cdot 50 \cdot 49\, x^{48}$
$\boxed{y^{50} = 3 \cdot 50!}$ C

(15) $y = \sqrt{x^2+16}$
$y' = \frac{1}{2}(x^2+16)^{-1/2}(2x)$
$= x(x^2+16)^{-1/2}$
$y'' = x\left(-\frac{1}{2}(x^2+16)^{-3/2}(2x)\right) + (x^2+16)^{-1/2}(1)$
$y' = \frac{-x^2}{(x^2+16)^{3/2}} + \frac{1}{(x^2+16)^{1/2}}$
$y' = \frac{-x^2+x^2+16}{(x^2+16)^{3/2}} = \boxed{\frac{16}{(x^2+16)^{3/2}}}$ E

(16) $f(x) = \log_3(3-2x)$ $\quad \log_a u \quad \frac{u'}{u \ln a}$
$f'(x) = \left(\frac{1}{\ln 3}\right)\left(\frac{1}{3-2x}\right)(-2)$ $\quad \frac{-2}{(3-2x)\ln 3}$
$f'(1) = \boxed{\frac{-2}{\ln 3}}$ B

(17) $y = \ln|\sec x - \tan x|$

$y' = \dfrac{\sec x \tan x - \sec^2 x}{\sec x - \tan x} = \dfrac{\sec x(\tan x - \sec x)}{\sec x - \tan x} = \sec x$

~~$\boxed{y' = -(\sec x \tan x)}$~~ D

(18) $f(x) = x^3 \ln x$

$f'(x) = x^3\left(\frac{1}{x}\right) + \ln x(3x^2)$

$\boxed{f'(x) = x^2 + \ln x\,(3x^2)}$ A

(19) $\frac{d}{dx}(\ln e^{2x})$

$\frac{d}{dx}\, 2x = \boxed{2}$ E

(20) $f(x) = \ln(\ln x)$

$f'(x) = \dfrac{\frac{1}{x}}{\ln x} = \dfrac{1}{x \ln x}$

$f'(e) = \dfrac{1}{e \ln e} = \boxed{\frac{1}{e}}$ C

(21) $y = x^{(x^3)}$

$\ln y = x^3 \ln x$

$\frac{y'}{y} = x^3\left(\frac{1}{x}\right) + \ln x(3x^2)$

$y' = (x^2 + 3x^2 \ln x)(x^{x^3})$

$\boxed{y' = (x^{x^3+2})(1 + 3\ln x)}$ A

(22) $y = e^{-x^2}$

$y' = e^{-x^2}(-2x)$

$y'' = e^{-x^2}(-2) + (-2x)(e^{-x^2})(-2x)$

$y''(0) = \boxed{-2}$ D

(23) $f(x)=\frac{1}{3}e^{3-2x}$
$f'(x)=\frac{1}{3}e^{3-2x}(-2)$
$=\boxed{\frac{-2}{3}e^{3-2x}}$ A

(24) $f(t)=\frac{e^{t}-e^{t/2}}{2}$
$f'(t)=\frac{1}{2}(e^{t}-\frac{1}{2}e^{\frac{t}{2}})=\frac{1}{2}e^{t}-\frac{1}{4}e^{t/2}$
$f''(t)=\frac{1}{2}e^{t}-\frac{1}{4}e^{t/2}(\frac{1}{2})$
$f''(0)=\frac{1}{2}-\frac{1}{8}=\boxed{3/8}$ D

(25) $\frac{d}{dx}\arctan 2x$ $u=2x$ $du=2$
$=\frac{1}{1+(2x)^2}(2)$
$=\boxed{\frac{2}{1+4x^2}}$ A

(26) $\frac{d}{dx}(\arcsin\frac{1}{x})$ $u=1/x$ $du=\frac{-1}{x^2}$
$=\frac{1}{\sqrt{1-(1/x)^2}}\left(-\frac{1}{x^2}\right)$
$=\frac{-1}{x^2\sqrt{1-(1/x)^2}}=\frac{-1}{x\sqrt{\frac{x^2-1}{x^2}}}$
$=\boxed{\frac{-1}{|x|\sqrt{x^2-1}}}$ A

(27) $2x^2-y^4=1$ $(1,1)$
$4x-4y^3\frac{dy}{dx}=0$
$4-4\frac{dy}{dx}=0$ $\frac{dy}{dx}=1$
$y-1=1(x-1)$
$y-1=x-1$
$\boxed{y=x}$ B

(28) $\sin x=\ln y$ $y=e^{\sin x}$
$\cos x=\frac{y'}{y}$
$\cos x\, y=y'$
$\boxed{\cos x(e^{\sin x})=y'}$ A

(29) $xy^2-3x+4y-2=0$
$x(2y)\frac{dy}{dx}+y^2-3+4\frac{dy}{dx}=0$
$(2xy+4)\frac{dy}{dx}=-y^2+3$
$\boxed{\frac{dy}{dx}=\frac{-y^2+3}{2xy+4}}$ D

(30) $\sin(xy)=y$
$\cos(xy)(x\frac{dy}{dx}+y)=\frac{dy}{dx}$
$x\cos(xy)\frac{dy}{dx}+y\cos(xy)=\frac{dy}{dx}$

$x\cos(xy)\frac{dy}{dx}-\frac{dy}{dx}=-y\cos(xy)$
$(x\cos(xy)-1)\frac{dy}{dx}=-y\cos(xy)$
$\frac{dy}{dx}=\frac{-y\cos(xy)}{x\cos(xy)-1}$
$\boxed{\frac{dy}{dx}=\frac{y\cos(xy)}{1-x\cos(xy)}}$ D