

# AP Calculus AB

## Set 8

### #1 (no calculator)

A particle moves along the x-axis so that its velocity at any time  $t \geq 0$  is given by  $v(t) = 1 - \sin(2\pi t)$ .

- Find the acceleration  $a(t)$  of the particle at any time  $t$ .
- Find all values of  $t$ ,  $0 \leq t \leq 2$ , for which the particle is at rest.
- Find the position  $x(t)$  of the particle at any time  $t$  if  $x(0) = 0$ .

### #2 (no calculator)

A particle moves along the x-axis so that its velocity at time  $t$ ,  $0 \leq t \leq 5$ , is given by  $v(t) = 3(t-1)(t-3)$ . At time  $t = 2$ , the position of the particle is  $x(2) = 0$ .

- Find the minimum acceleration of the particle.
- Find the total distance traveled by the particle.
- Find the average velocity of the particle over the interval  $0 \leq t \leq 5$ .

### #3 (no calculator)

A particle moves along the x-axis so that at any time  $t > 0$  its velocity is given by  $v(t) = t \ln t - t$ .

- Write an expression for the acceleration of the particle.
- For what values of  $t$  is the particle moving to the right?
- What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

### #4 (no calculator)

A particle moves along the x-axis with velocity at time  $t \geq 0$  given by  $v(t) = -1 + e^{t-3}$ .

- Find the acceleration of the particle at time  $t = 3$ .
- Is the speed of the particle increasing at time  $t = 3$ ? Give a reason for your answer.
- Find all values of  $t$  at which the particle changes directions. Justify your answer.
- Find the total distance traveled by the particle over the time interval  $0 \leq t \leq 3$ .

**#5 (calculator)**

A particle moves along the  $x$ -axis so that its velocity  $v$  at any time  $t$ , for  $0 \leq t \leq 16$ , is given by  $v(t) = e^{2 \sin t} - 1$ . At time  $t = 0$ , the particle is at the origin.

- On the axes provided, sketch the graph of  $v(t)$  for  $0 \leq t \leq 16$ .
- During what intervals of time is the particle moving to the left? Give a reason for your answer.
- Find the total distance traveled by the particle from  $t = 0$  to  $t = 4$ .
- Is there any time  $t$ ,  $0 \leq t \leq 16$ , at which the particle returns to the origin? Justify your answer.

**#6 (calculator)**

A tank contains 125 gallons of heating oil at time  $t = 0$ . During the time interval  $0 \leq t \leq 12$  hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate  $R(t) = 12 \sin\left(\frac{t^2}{47}\right)$  gallons per hour.

- How many gallons of heating oil are pumped into the tank during the time interval  $0 \leq t \leq 12$  hours?
- Is the level of heating oil in the tank rising or falling at time  $t = 6$  hours? Give a reason for your answer.
- How many gallons of heating oil are in the tank at time  $t = 12$  hours?
- At what time  $t$ , for  $0 \leq t \leq 12$ , is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

**#7 (calculator)**

A particle moves along the  $x$ -axis so that its velocity  $v$  at time  $t$ , for  $0 \leq t \leq 5$ , is given by

$$v(t) = \ln(t^2 - 3t + 3). \text{ The particle is at position } x = 8 \text{ at time } t = 0.$$

- Find the acceleration of the particle at time  $t = 4$ .
- Find all times  $t$  in the open interval  $0 < t < 5$  at which the particle changes direction. During which time intervals, for  $0 \leq t \leq 5$ , does the particle travel to the left?
- Find the position of the particle at time  $t = 2$ .
- Find the average speed of the particle over the interval  $0 \leq t \leq 2$ .

18 (calculator)

A water tank at Camp Newton holds 1200 gallons of water at time  $t = 0$ . During the time interval  $0 \leq t \leq 18$  hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t} \sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275 \sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

- (a) Is the amount of water in the tank increasing at time  $t = 15$ ? Why or why not?
  - (b) To the nearest whole number, how many gallons of water are in the tank at time  $t = 18$ ?
  - (c) At what time  $t$ , for  $0 \leq t \leq 18$ , is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.
  - (d) For  $t > 18$ , no water is pumped into the tank, but water continues to be removed at the rate  $R(t)$  until the tank becomes empty. Let  $k$  be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of  $k$ .
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