AP Calculus AB – 1st semester practice multiplechoice.

**Limits**

1. $\lim_{x\to -2} \dfrac{-3x-6}{x^2+x-2}$ is

A. -1 B. $\dfrac{-1}{3}$ C. $\dfrac{1}{3}$ D. 1 E. Does not exist

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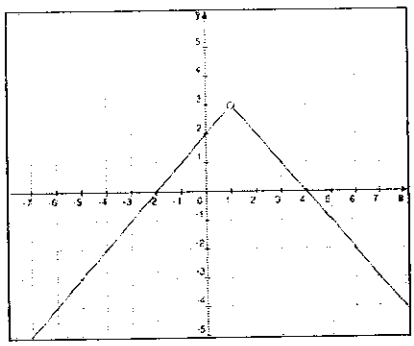
2. The figure to the right shows the graph of $f(x)$.
Which of the following statements are true?



I. $\lim_{x\to 1^-} f(x)$ exists

II. $\lim_{x\to 1^+} f(x)$ exists

III. $\lim_{x\to 1} f(x)$ exists

A. I only B. II only C. I and II only D. I, II and III E. none are true

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3. Let $f(x)=\begin{cases} \dfrac{x^2-6x+9}{x-3} & \text{if } x \neq 3 \\ \sqrt{3x} & \text{if } x = 3 \end{cases}$ Which of the following statements I,II, and III are true?

I. $\lim_{x\to 3} f(x)$ exists II. $f(3)$ exists III. $f$ is continuous at $x=3$

A. only I B. only II C. I and II D. none of them E. all of them

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4. $\lim_{x\to -\infty} \dfrac{-4-x-x^2}{2x^2+3x-2} =$ is

A. 0 B. 2 C. -2 D. $\dfrac{-1}{2}$ E. $\infty$

5. If $f(x)=\left(1-4x^2\right)^4$, then $f'(x)=$

A. $32x\left(1-4x^2\right)^3$ B. $4(1-8x)^3$ C. $-32(1-8x)^3$ D. $4\left(1-4x^2\right)^3$ E. $-32x\left(1-4x^2\right)^3$

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6. The slope of the tangent line to $y=\dfrac{x}{2x-1}$ at $x=4$ is

A. $\dfrac{1}{2}$ B. $\dfrac{-1}{14}$ C. $\dfrac{1}{-14}$ D. $\dfrac{1}{49}$ E. $\dfrac{-1}{49}$

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7. The second derivative of $f(x)=\sin x\cos x$ is

A. 0 B. $-4\sin x\cos x$ C. $(\cos x)^2-(\sin x)^2$
D. $2\cos x-2\sin x$ E. $2\sin x-2\cos x$

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8. If $f(-2)=4$ and $f'(-2)=-1$, find the derivative of $\dfrac{f(x)}{x^2}$ at $x=-2$.

A. $\dfrac{3}{4}$ B. 1 C. $\dfrac{1}{4}$ D. $-\dfrac{1}{4}$ E. $\dfrac{1}{2}$

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9. Find the equation of the line tangent to $y=\sec x$ at $x=\dfrac{\pi}{4}$.

A. $y-\dfrac{\sqrt{2}}{2}=\sqrt{2}\left(x-\dfrac{\pi}{4}\right)$ B. $y-\sqrt{2}=\sqrt{2}\left(x-\dfrac{\pi}{4}\right)$ C. $y=\sqrt{2}\left(x-\dfrac{\pi}{4}\right)$
D. $y-\sqrt{2}=x-\dfrac{\pi}{4}$ E. $y-\sqrt{2}=-\left(x-\dfrac{\pi}{4}\right)$

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10. For how many values of $x$ are the tangent lines to $y=\sin x+\cos x$ and $y=\dfrac{x^3}{12}-\dfrac{x^2}{2}-\dfrac{x}{4}$ parallel?

A. 0 B. 1 C. 2 D. 3 E. 4

11. Let $f$, $g$ and their derivatives be defined by the table below. If the derivatives of $f(g(x))$ and $g(f(x))$ are equal at $x = 3$, what is the value of $a$?

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$ | 3 | 2 | 1 | 4 |
| $g(x)$ | 2 | 1 | 4 | 3 |
| $f'(x)$ | 4 | 3 | 4 | 2 |
| $g'(x)$ | $a$ | 1 | 2 | 3 |

A. 1 B. 2 C. 3 D. 4 E. Impossible

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**Implicit Differentiation**

12. If $x^2 + xy - y = 7$, find $\frac{dy}{dx}$ at $(3,-1)$

A. -3 B. 1 C. $-\frac{5}{2}$ D. -1 E. $\frac{1}{2}$

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13. At which points is the tangent line to the curve $8x^2 + 2y^2 = 6xy + 14$ vertical?

I. (-2, -3) II (3, 8) III. (4, 6)

A. I only B. II only C. III only D. I and II only E. I and III only

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**Continuity and Differentiability**

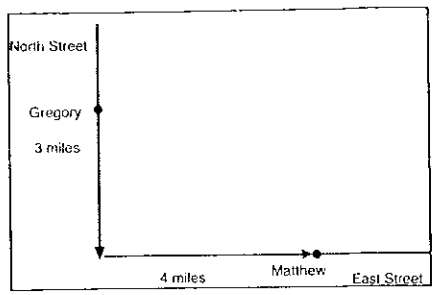
14. If $f(x) = \begin{cases} x^2 - 5x + 8, x \geq 3 \\ x - 1, x < 3 \end{cases}$, describe the behavior of the graph of $f(x)$ at $x = 3$

I. $\lim_{x \to 3} f(x)$ exists II. $f(x)$ is continuous at $x = 3$ III. $f(x)$ is differentiable at $x = 3$

a) I only b) II only c) I and II only d) I, II and III e) none are true

**Related Rates**

15. Matthew is visiting Gregory at his home on North Street. Shortly after Matthew leaves, Gregory realizes that Matthew left his wallet and begins to chase him. When Gregory is 3 miles from the 90° intersection along North Street traveling at 45 mph towards the intersection, Matthew is 4 miles along East street traveling away from the intersection at 30 mph. At that time, how fast is the distance between the two men changing?




A. getting closer at 3 mph
B. getting further away at 51 mph
C. getting closer at 51 mph
D. Getting closer at 15 mph
E. getting closer at 18 mph

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16. A cylinder has both its height and radius changing. Its height is increasing at the rate of 3 meters/min. When the height of the cylinder is 8 meters and its radius is 2 meters, the volume is not changing. How fast is the radius decreasing in meters/min? (The volume of a cylinder is given by $V = \pi r^2 h$).

A. 3 B. $\frac{3}{8}$ C. 6 D. $3\pi$ E. $6\pi$

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**Function Analysis**

17. Given that $f(x) = x^3$ find all values of $x$ in the interval $(-1,1)$ that satisfy the mean value theorem.

A. 0 B. $\sqrt{\frac{1}{3}}$ C. $\pm\sqrt{\frac{1}{3}}$ D. 1 E. $\pm -1$

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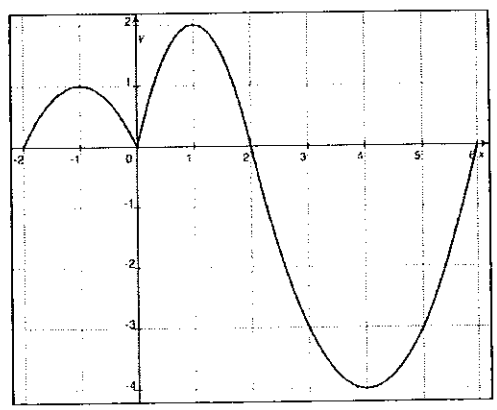
18. The graph of $f'$, the derivative of $f$ is shown to the right for $-2 \le x < 6$. At what values of $x$ does $f$ have a horizontal tangent line?



A. $x = 0$ only
B. $x = -1, x = 1, x = 4$
C. $x = 2$ only
D. $x = -2, x = 2, x = 6$
E. $x = -2, x = 0, x = 2, x = 6$

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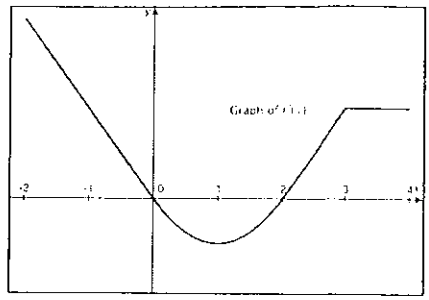
19. Given $f(x) = 2 - \frac{x^3}{6} - x^2$. On what interval(s) is the graph of $f$ concave upwards?

A. $(-\infty,-2)$ B. $(-\infty,2)$ C. $(-2,\infty)$ D. $(2,\infty)$ E. $(-2,2)$

20. The graph of $f'(x)$, the derivative of $f$, is shown to the right. Which of the following statements is **not** true?



A. $f$ is increasing on $2 < x \le 4$.
B. $f$ has a local minimum at $x = 1$.
C. $f$ has a local maximum at $x = 0$.
D. $f$ has an inflection point at $x = 1$.
E. $f$ is concave down on $-2 \le x < 1$.

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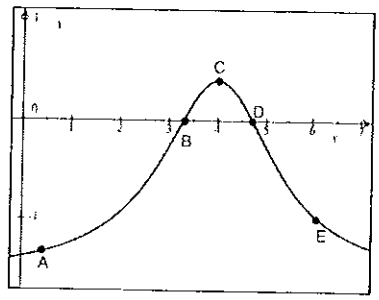
21. The graph of $y = f(x)$ is to the right. At what point are both $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} > 0$ ?

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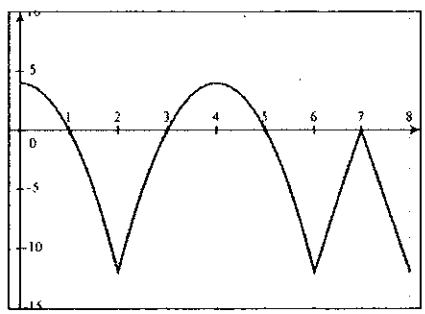
22. If $f(x) = \frac{1}{x^n}$, where $n > 0$ and $x > 0$, describe the concavity of the graph of $f(x)$.

A. Always concave up
B. Always concave down
C. Concave down if $n < 1$, concave up if $n \ge 1$
D. Concave up if $n < 1$, concave down if $n \ge 1$
E. Concave up if $n < 2$, concave down if $n \ge 2$

**Straight-Line Motion**

23. A locomotive is moving along a straight track. Its velocity $v$ of the locomotive at time $t, 0 \le t \le 8$ is given by the function whose graph is to the right. At what value of $t$ does the locomotive change direction?




A. 4 only B. 2 and 4 only C. 2, 4 and 6 only

D. 1, 3, and 5 only E. 1, 3, 5 and 7 only

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24. A particle moves along a horizontal line with position $x(t) = \frac{10}{t}$. Describe its motion at $t = 1$.

A. Moving right and slowing down
B. Moving right and speeding up
C. Moving left and slowing down
D. Moving left and speeding up
E. Stopped

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25. A ball is thrown straight up from the top of a hill 30 feet high with initial velocity of 72 ft/sec. How high above level ground will the ball get? (objects subjected to gravity adhere to $s(t) = -16t^2 + v_0t + s_0$ where $s$ is the height of the object in feet, $v_0$ is the initial velocity and $s_0$ is the initial height).

A. 72 sec B. 81 feet C. 88 feet D. 111 ft E. 144 ft

**Optimization**

26. If $f(x)=\frac{x^3}{3}-x^2$ on $[-2,3]$, $f(x)$ has an absolute minimum at

A. $x=-2$ B. $x=0$ C. $x=2$ D. $x=3$ E. $x=0$ and $x=2$

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27. If the sum of two numbers $x$ and $y$ is 12, what is the maximum product of $x^3y$?

A. 9 B. 27 C. 729 D. 2187 E. 19683

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28. $f(x)=\frac{x^2}{4}+x-4\cos(2x)$ on $[0, 2\pi]$ has a possible maximum slope at the $x$-value that solves the equation

A. $\frac{x}{2}+1-8\sin(2x)=0$ B. $\frac{x}{2}+1+8\sin(2x)=0$ C. $\frac{x}{2}+1+4\sin(2x)=0$

D. $\frac{1}{2}+16\cos(2x)=0$ E. $\frac{1}{2}+4\cos(2x)=0$

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**Indefinite Integration**

29. $\int\left(x^2-3x+1-5\sin x\right)dx$

A. $\frac{x^3}{3}-\frac{3x^2}{2}-5\cos x+C$ B. $\frac{x^3}{3}-\frac{3x^2}{2}+5\cos x+C$ C. $\frac{x^3}{3}-\frac{3x^2}{2}+x-5\cos x+C$

D. $\frac{x^3}{3}-\frac{3x^2}{2}+x+5\cos x+C$ E. $2x-3-5\cos x+C$

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30. If $f'(x)=\left(x^4-x\right)^2$ and $f(1)=1$, find $f(x)$

A. $\frac{x^9}{9}+\frac{x^3}{3}+\frac{5}{9}$ B. $\frac{x^9}{9}-\frac{x^6}{6}+\frac{x^3}{3}+\frac{13}{18}$ C. $\frac{x^9}{9}-\frac{x^6}{3}+\frac{x^3}{3}+\frac{8}{9}$

D. $\frac{x^9}{9}-\frac{x^6}{3}+\frac{x^3}{3}+\frac{1}{9}$ E. $\frac{x^9}{9}-\frac{x^6}{3}+\frac{x^3}{3}+1$