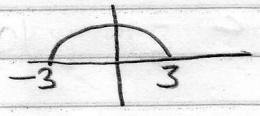
AP Calculus

Midterm Exam Review #2

1) $\lim_{x\to -1^+} = \infty$ $\lim_{x\to -1^-} = -\infty$ therefore $\lim_{x\to -1}$ DNE (D)

2) $\lim_{x\to -1} \frac{x^2+2x+3}{x^2+1}$ plug in $-1 \to \frac{2}{2} = 1$ (B)

3) $\lim_{x\to 3} \sqrt{9-x^2}$  $\lim_{x\to 3}$ DNE from right (D)

4) $\lim_{x\to c}[f(x)g(x)] = \lim_{x\to c}(-1/2)(2/3) = -1/3$ (B)

5) $\lim_{x\to -1} \frac{x^2-5x-6}{x+1} = \frac{(x-6)(x+1)}{x+1}$ hole at $x=-1$

$\lim_{x\to -1} x-6 = -7$ (B)

6) $2x-3y=3 \to -3y = -2x+3 \to y = \frac{2}{3}x - 1$ $(4,-1)$

$y+1 = 2/3(x-4)$

$3y+3 = 2x-8 \to 2x-3y = 11$ (A)

7) $x=7$ m = undefined $\perp$ $m=0$ $(1,-4)$

$y+4 = 0(x-1) \to y+4=0$ (A)

8) $f(x) = \frac{1}{\sqrt{3+2x}}$ $3+2x > 0$, $2x > -3$, $x > -3/2$ domain $(-3/2, \infty)$ (C)

9) def. of derivative $\lim_{\Delta x\to 0} \frac{f(x+h)-f(x)}{\Delta x}$ (C)

10) $y = \frac{1+x^2}{1-x^2}$ quotient rule $y' = \frac{(1-x^2)(2x)-(1+x^2)(-2x)}{(1-x^2)^2}$

$y' = \frac{2x-2x^3+2x+2x^3}{(1-x^2)^2} = \frac{4x}{(1-x^2)^2}$ (E)

11) $y = x^3\sqrt{x+1}$ $x^3(x+1)^{1/2}$ product rule $y' = x^3\left(\frac{1}{2}\right)(x+1)^{-1/2} + (x+1)^{1/2}3x^2$

$y' = \frac{x^3}{2\sqrt{x+1}} + \frac{3x^2\sqrt{x+1}}{1}$

$y' = \frac{x^3+6x^2(x+1)}{2\sqrt{x+1}} = \frac{7x^3+6x^2}{2\sqrt{x+1}}$

$= x^2(7x+6)/2\sqrt{x+1}$ (B)

(12) $f(x) = (2x^2+5)^7$

$f'(x) = 7(2x^2+5)^6(4x)$

$f'(x) = 28x(2x^2+5)^6$ (C)

(13) $y = \frac{x+3}{x-1}$

$y' = \frac{(x-1)-(x+3)}{(x-1)^2} = \frac{-4}{(x-1)^2}$

$y' = -4(x-1)^{-2}$

$y'' = 8(x-1)^{-3} = \frac{8}{(x-1)^3}$ (D)

(14) $S = (t^2-1)^3$ acceleration → 2nd deriv

$S' = 3(t^2-1)^2(2t)$ chain rule

$S' = 6t(t^2-1)^2$ product / chain rule

$S'' = 6t(2)(t^2-1)(2t) + (t^2-1)^2(6)$

$S'' = 12t(t^2-1)(2t) + 6(t^2-1)^2$

$S''(2) = 24(3)(4) + 54 = 342$ (A)

(15) $y = 2x^2 + 8x$ $[0,3]$

$y' = 4x + 8 = 0$ $x = -2$

$y(0) = 0$

$y(-2) = -8$ min (E)

$y(3) = 42$ max

(16) $y^2 - 3xy + x^2 = 7$

$2y\frac{dy}{dx} - 3x\frac{dy}{dx} + y(-3) + 2x = 0$

$(2y-3x)\frac{dy}{dx} = 3y - 2x$

$\frac{dy}{dx} = \frac{3y-2x}{2y-3x}$ (B)

(17) $y = \frac{1}{x+y}$

$\frac{dy}{dx} = (x+y)(0) - 1\left(1+\frac{dy}{dx}\right)$

$\frac{dy}{dx} = -1 - \frac{dy}{dx}$

$2\frac{dy}{dx} = -1 \rightarrow \frac{dy}{dx} = -\frac{1}{2}$ (E)

(18) $f(x) = \frac{3}{\sqrt[3]{x^2}} = 3x^{-2/3}$

$f'(x) = -2x^{-5/3}$

$= \frac{-2}{x^{5/3}}$ (B)

(19) $y = \frac{3x}{\sqrt{x+1}}$ quotient

$y' = \frac{\sqrt{x+1}(3) - 3x\left(\frac{1}{2}\right)(x+1)^{-1/2}}{(\sqrt{x+1})^2}$

$y' = \frac{3\sqrt{x+1} - \frac{3x}{2\sqrt{x+1}}}{x+1}$

$y' = \frac{6(x+1)-3x}{2\sqrt{x+1}} = \frac{3x+6}{2\sqrt{x+1}} \cdot \frac{1}{x+1}$

$y' = \frac{3(x+2)}{2(x+1)^{3/2}}$ (A)

(20) $f(x) = 2x^2 - 2x + 3$ $x=1$, $y=3$

$f'(x) = 4x - 2$

$f'(1) = 2$ (slope)

$y - 3 = 2(x-1)$

$y - 3 = 2x - 2$

$y = 2x + 1$ (C)

(21) $f(x) = -x^3 + 3x^2 - 2$ $m = 0$

$f'(x) = -3x^2 + 6x = 0$

$-3x(x-2) = 0$

$x = 0, 2$ $(0,-2)(2,2)$

(A)

(22) $w = \frac{7}{3z^2} = \frac{7}{3}z^{-2}$

$w' = -\frac{14}{3}z^{-3} = \frac{-14}{3z^3}$

(D)

(23) $s(t) = 3t^2 + 2t + 5$

$v(t) = s'(t) = 6t + 2$

$v(2) = 14$ (B)

(24) $f(x) = \frac{1}{(x-2)^2}$ VA $x=2$, HA $y=0$

$f(x) = (x-2)^{-2}$

$f'(x) = -2(x-2)^{-3} = \frac{-2}{(x-2)^3}$

$f'(x)$: + on $(-\infty, 2)$, − on $(2, \infty)$

inc $(-\infty, 2)$

dec $(2, \infty)$ (B)

(25) $y = x^3 - 12x + 20$

$y' = 3x^2 - 12 = 0$

$3x^2 = 12 \to x^2 = 4$

$x = \pm 2$

$f'(x)$: + on $(-\infty, -2)$, − on $(-2, 2)$, + on $(2, \infty)$

max $(-2, 36)$ min $(2, 4)$

inc $(-\infty, -2)(2, \infty)$

min $(-2, 2)$ (A)

(26) $f(x) = \frac{x}{x^2+4}$

$f'(x) = \frac{(x^2+4) - x(2x)}{(x^2+4)^2}$

$f'(x) = \frac{x^2+4-2x^2}{(x^2+4)^2} = \frac{-x^2+4}{(x^2+4)^2}$

$-x^2 + 4 = 0 \to -x^2 = -4 \to x^2 = 4$

$x = \pm 2$

$(x^2+4)^2 = 0 \to x^2 = -4$

$f'(x)$: − on $(-\infty, -2)$, + on $(-2, 2)$, − on $(2, \infty)$

dec $(-\infty, -2)(2, \infty)$ (E)

(27) $f(x) = \frac{x-1}{x+3}$ $x=-3$ VA

$f'(x) = \frac{(x+3)-(x-1)}{(x+3)^2} = \frac{4}{(x+3)^2}$ $x \neq -3$ (C)

(28) $f(x) = 3x^5 - 5x^3$

$f'(x) = 15x^4 - 15x^2 = 0$

$15x^2(x^2-1) = 0$

$x = 0, 1, -1$

$f'(x)$: + on $(-\infty, -1)$, − on $(-1, 0)$, − on $(0, 1)$, + on $(1, \infty)$

max $x = -1$ min $x = 1$ (B)

(29) $f(x) = \frac{x^2+1}{x^2} = 1 + x^{-2}$

$f'(x) = -2x^{-3}$

$f''(x) = 6x^{-4} = \frac{6}{x^4}$ $x \neq 0$

f'': + on $(-\infty, 0)$, + on $(0, \infty)$ (C)

(30) $f''(x) = 4x^3 - 2x = 0$ (A)

$f''(-1) = -2$ $f(x)$ ccd ∩ max

$f''(0) = 0$

$f''(1) = 2$ $f(x)$ ccu ∪ min

(31) $\lim_{x \to \infty} \frac{2x^3 + 6x^2 + 5}{3 + x^3}$

degree num = deg denom

$\lim_{x \to \infty} = 2$ (D)

(32) horiz $y = 2$ (C)

(33) $f(x) = \frac{1}{12}x^4 - 2x^2 + 15$

$f'(x) = \frac{1}{3}x^3 - 4x$

$f''(x) = x^2 - 4 = 0$

$x = \pm 2$

$f''(x)$: + on $(-\infty, -2)$, − on $(-2, 2)$, + on $(2, \infty)$

POI $(-2, 20/3)(2, 20/3)$ (D)

(34) omit

(35) $f(x) = \frac{x-4}{(x-2)(x+1)}$

VA $x = 2, -1$ (C)

(36) $f(x) = \frac{1}{\sqrt{x}}$ $g(x) = x-1$

$f(g(x)) = \frac{1}{\sqrt{x-1}}$

$x-1 > 0$ $x > 1$ domain

$fg(x)$ discontinuous $x \leq 1$ (E)

(37) $f(x) = \begin{cases} x+3 & x \leq -1 \\ 2x-c & x > -1 \end{cases}$

$f(-1) = 2$

$\lim_{x \to -1^+} 2(-1) - c$ $\lim_{x \to -1^-} -1+3 = 2$

$-2-c = 2$ $c = 4$ (B)

$-c = 4$

(38) $f(x) = \frac{2x-1}{x+3}$

VA $x = -3$ (C)

(39) $f(x) = \frac{x-2}{x^2-4} = \frac{x-2}{(x+2)(x-2)}$ hole

VA $x = -2$ (B)

(40) Skip

(41) $\lim_{x \to 0^+} \frac{1}{x} = \frac{+}{+} = \infty$ (A)

(42) $\lim_{x \to 1} \frac{5}{(x-1)^2} = \frac{5}{0} = \frac{+}{+} = \infty$ (D)

(43) $\lim_{x \to 1} \left(2 - \frac{5}{(x-1)^2}\right) = \frac{2(x-1)^2 - 5}{(x-1)^2} = \frac{-5}{0} = \frac{-}{+} = -\infty$