Chapter 5 Review

$\frac{1}{2}x$

① $y = \arcsin \frac{x}{2}$ $(0,0)$

$$y' = \frac{u'}{\sqrt{1-u^2}} = \frac{1/2}{\sqrt{1-(\frac{x}{2})^2}} = \frac{1/2}{1} = \boxed{1/2}$$

$$y - 0 = 1/2(x-0)$$
$$y = 1/2 x \qquad 2y = x \qquad \boxed{x - 2y = 0}$$

A

② $f(x) = x^2 + e^{-2x}$

$f'(x) = 2x + e^{-2x}(-2)$

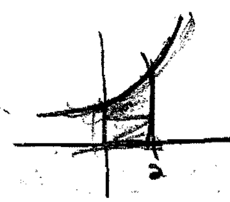
@ 0 $\quad f'(0) = 0 + e^0(-2)$

$= -2$ derivative negative →

f decreasing

B

③ $y = e^{2x}$ x-axis, y-axis, $x = 2$



$$A = \int_0^2 (e^{2x} - 0)\,dx = \frac{1}{2} e^{2x} \Big|_0^2$$
$$\frac{1}{2}(e^4 - e^0) = \boxed{\frac{1}{2}(e^4 - 1)}$$

C

④ $\frac{dy}{dx} = \tan x\,dx \qquad \int \tan x = -\ln|\cos x| + C$

$= \ln|\cos x|^{-1} + C$

$= \boxed{\ln|\sec x| + C}$

C

⑤ $y = \arctan(e^{2x})$

$$y' = \frac{u'}{1+u^2} = \frac{2e^{2x}}{1+(e^{2x})^2} = \boxed{\frac{2e^{2x}}{1+e^{4x}}}$$

B

(6) $y = \ln(x^2)$ $\quad x = e^2$

$y' = \frac{2x}{x^2} = \frac{2}{x} \rightarrow \boxed{\frac{2}{e^2}}$ (B)

(7) $x=0,\ x=2,\ y=0 \quad y = e^{x/2}$ $\quad u = \frac{x}{2} \quad du = \frac{1}{2}$

$A = \int_0^2 e^{x/2}\,dx \rightarrow 2\int e^u\,du = 2e^{x/2}\Big|_0^2$

$2(e^1 - e^0) = \boxed{2(e-1)}$ (C)

(8) $d/dx\,(\arcsin(2x)) = \frac{u'}{\sqrt{1-u^2}} = \frac{2}{\sqrt{1-(2x)^2}} = \boxed{\frac{2}{\sqrt{1-4x^2}}}$ (D)

(9) $\int_0^1 (x+1)e^{x^2+2x}\,dx$ $\quad u = x^2+2x$
$du = 2x+2 = 2(x+1)\,dx$

$\frac{1}{2}\int e^u\,du = \frac{1}{2}e^{x^2+2x}\Big|_0^1 = \frac{1}{2}(e^3 - e^0)$ (B)
$= \boxed{\frac{1}{2}(e^3-1)}$

(10) $f(x) = e^{1/x}$ $\quad x^{-1}$
$f'(x) = e^{1/x}(-1x^{-2}) = \boxed{\frac{-e^{1/x}}{x^2}}$ (A)

(11) $y = 10^{(x^2-1)}$ $\quad a^u = a^u u' \ln a$
$\boxed{y' = 10^{(x^2-1)}(2x)(\ln 10)}$ (D)

(12) $\int \tan(2x)\,dx = \frac{1}{2}\int \tan u\,du = \boxed{-\frac{1}{2}\ln|\cos 2x| + C}$
$u = 2x$
$du = 2$
(B)

(13) $\int \frac{1}{\sqrt{25-x^2}}\,dx = \frac{1}{\sqrt{a^2-u^2}} = \boxed{\arcsin\frac{x}{5}+C}$

$a^2 \quad u^2$

$\arcsin\frac{u}{a}$

(A)

(14) $\int_0^1 x^3 e^{x^4}\,dx \quad u=x^4 \quad du = 4x^3$

$\frac{1}{4}\int e^u\,du = \frac{1}{4}e^{x^4}\Big|_0^1 = \boxed{\frac{1}{4}(e-1)}$ (A)

(15) $f(x) = x^2e^x$

$f'(x) = x^2e^x + e^x 2x = 0$

$xe^x(x+2) = 0$

$x=0 \quad e^x=0 \quad x+2=0$

$x=0$, und, -2

$+ \quad - \quad +$ at -2, 0

$\boxed{\text{decreasing } -2<x<0}$ (B)

(16) $y = \frac{\ln x}{x}$

$y' = \frac{x\left(\frac{1}{x}\right) - \ln x(1)}{x^2} = \boxed{\frac{1-\ln x}{x^2}}$ (D)

(17) $\int_2^3 \frac{x}{x^2+1} \quad u = x^2+1 \quad du = 2x$

$\frac{1}{2}\int_2^3 \frac{1}{u}\,du = \frac{1}{2}\ln|x^2+1|\Big|_2^3$

$= \frac{1}{2}(\ln 10 - \ln 5) = \frac{1}{2}\ln\frac{10}{5}$

$\boxed{\frac{1}{2}\ln 2}$ (B)

(18) ~~$y = e^{2x}$, $x = 2x + 5$, $y = 2x + 5$, $x = 2$ repeat~~

(19) $\frac{d}{dx}(\ln e^{2x}) = \frac{d}{dx} 2x = 2$ (E)

(20) $f(x) = \int_1^x \frac{1}{t}\,dt = \boxed{\frac{1}{x}}$ (B) $f(x) = \int f'(x)$

(21) ~~repeat~~

ln rules - shortcut

(22) $\frac{d}{dx} \ln\left(\frac{1}{1-x}\right) = \frac{d}{dx}(\cancel{\ln 1} - \ln(1-x)) = -\frac{-1}{1-x} = \boxed{\frac{1}{1-x}}$ (A)

$\frac{u'}{u}$ or long way $\ln\left(\frac{1}{1-x}\right) = \ln(1-x)^{-1}$

$\frac{d}{dx} = \frac{\cancel{-1}(1-x)^{-2}(\cancel{-1})}{(1-x)^{-1}} = \frac{1}{1-x}$

(23) $\frac{1}{3-1}\int_1^3 \frac{1}{x}\,dx = \frac{1}{2}\ln|x|\Big|_1^3 = \frac{1}{2}(\ln 3 - \ln 1)$

$= \frac{1}{2}\ln 3$ (D)

(24) $\int \frac{5}{1+x^2}\,dx = 5\int \frac{1}{\underset{a^2+u^2}{1+x^2}} = 5 \cdot \frac{1}{a}\arctan\frac{u}{a} + C$

(D) $5\arctan x + C$

(25) $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x}\,dx = \int \cot x\,dx = \ln|\sin x|\Big|_{\pi/4}^{\pi/2} =$

$\ln\left(\sin\frac{\pi}{2}\right) - \ln\left(\sin\frac{\pi}{4}\right)$

$\cancel{\ln 1} - \ln \frac{\sqrt{2}}{2} = \ln\left(\frac{\sqrt{2}}{2}\right)^{-1} = \ln\frac{2}{\sqrt{2}}$ (A)

$= \boxed{\ln\sqrt{2}}$

$\boxed{\text{OR}}$

$\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x}\,dx$ $u = \sin x$
$du = \cos x\,dx$

$= \int \frac{1}{u}\,du = \ln|\sin x|\Big|_{\pi/4}^{\pi/2}$