Key

Calculus AB - Derivatives

Match each derivative

1) $f(x)=\tan x$ d
2) $f(x)=\sec x$ c
3) $f(x)=\csc x$ b
4) $f(x)=\sin x$ f
5) $f(x)=\cos x$ a
6) $f(x)=\cot x$ e

a) $f'(x)=-\sin x$
b) $f'(x)=-\csc x\cot x$
c) $f'(x)=\sec x\tan x$
d) $f'(x)=\sec^2 x$
e) $f'(x)=-\csc^2 x$
f) $f'(x)=\cos x$

---

7) $f(x)=3x^2-7x+8$

a) $f'(x)=$ $6x-7$

b) $f'(2)=$ 5

c) Equation of the tangent line at $x=2$ $(2,6)$ $y-6=\boxed{5}(x-2)$

d) Equation of the normal line at $x=2$ $y-6=-\frac{1}{5}(x-2)$

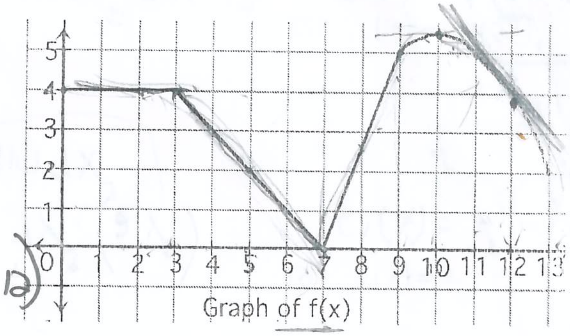
Use picture at right for #8

8 a) $f'(2)=$ 0 b) $f'(3)=$ DNE c) $f'(5)=$ -1

d) $f'(8)=$ 5/2 e) $f'(9)=$ DNE f) $f'(10)=$ 0

g) $f'(12)\doteq$ - h) Equation of tangent line at 12 $(12,4)$ $y-4=\square(x-12)$

equation of curve



Graph of f(x)

---

9) $\lim\limits_{h\to 0}\dfrac{\tan(x+h)-\tan x}{h}=$ $f(x)=\tan x$ $f'(x)=\sec^2 x$

10) $\lim\limits_{h\to 0}\dfrac{3(x+h)^7-3x^7}{h}=$ $f(x)=3x^7$ $f'(x)=21x^6$

11) $\lim\limits_{h\to 0}\dfrac{(1+h)^7-1}{h}=$ $f(x)=x^7$ at $x=1$ $f'(x)=7x^6$ $f'(1)=7$

12) $\lim\limits_{h\to 0}\dfrac{\cos\left(\frac{\pi}{6}+h\right)-\frac{\sqrt{3}}{2}}{h}=$ $f(x)=\cos x$ at $x=\pi/6$ $f'(x)=-\sin x$ $-\sin(\pi/6)=-1/2$

implicit

Find equation of the tangent line and normal line to the given equation at the given point.

13) $2xy^2+x=12$; $(4,-1)$

$y+1=3/16(x-4)$
$y+1=-16/3(x-4)$

$2x(2y)\frac{dy}{dx}+y^2 2+1=0$
$4xy\frac{dy}{dx}=-2y^2-1$
$dy/dx=\dfrac{-2y^2-1}{4xy}=\dfrac{-3}{-16}=\dfrac{3}{16}$

tangent line $y+1=3/16(x-4)$

normal line $y+1=-16/3(x-4)$

14) Given: $x^2-6x+y^2+4y-12=0$ and $\dfrac{dy}{dx}=\dfrac{-2x+6}{2y+4}$ $-\dfrac{2(x-3)}{2(y+2)}$

a) Find the horizontal tangent(s).

$-2x+6=0$
$-2x=-6$
$\boxed{x=3}$

b) Find the vertical tangent(s).

$2y+4=0$
$2y=-4$
$\boxed{y=-2}$

$2x - 18y\, dy/dx = 0$

$-18y\, dy/dx = -2x$

$\boxed{dy/dx = \frac{x}{9y}}$

15) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. $x^2 - 9y^2 = 7$

$$\frac{9y(1) - x\left(9\frac{dy}{dx}\right)}{(9y)^2} = \frac{9y - 9x\left(\frac{x}{9y}\right)}{81y^2} = \frac{9y - \frac{x^2}{y}}{81y^2} = \frac{9y^2 - x^2}{y} \cdot \frac{1}{81y^2}$$

$$= \boxed{\frac{9y^2 - x^2}{81y^3}}$$

$f(3) = 3$	$f'(3) = 7$	$f(9) = -2$	$f'(9) = 3$
$g(3) = -1$	$g'(3) = 4$	$g(9) = 0$	$g'(9) = 6$

16) Use information above to find $h'(x)$ and $h'(3)$.

a) $h(x) = f(x) \cdot g(x)$

$h'(x) = f(x)g'(x) + g(x)f'(x)$

$h'(3) = f(3)g'(3) + g(3)f'(3)$

$= (3)(4) + (-1)(7)$

$12 - 7$

5

b) $h(x) = g(f(x))$

$h'(x) = g'(f(x))f'(x)$

$h'(3) = g'(f(3))f'(3)$

$= g'(3)(7)$

$= (4)(7)$

$= 28$

c) $h(x) = (f(3x))^3$

$h'(x) = 3(f(3x))^2 f'(3x)(3)$

$h'(3) = 3(f(9))^2 f'(9)(3)$

$= 12(3)(3)$

$= 108$

Find derivatives for each.

17) $f(x) = \frac{2}{x} = 2x^{-1}$

$f'(x) = -2x^{-2}$

$= \boxed{\frac{-2}{x^2}}$

18) $f(x) = \cos 7x^3$

$f'(x) = -\sin 7x^3 (21x^2)$

$= \boxed{-21x^2 \sin 7x^3}$

19) $f(x) = x^{7/9}$

$f'(x) = \frac{7}{9}x^{-2/9}$

$= \boxed{\frac{7}{9x^{2/9}}}$

20) $f(x) = \tan(\sec x)$

$\boxed{f'(x) = \sec^2(\sec x)(\sec x \tan x)}$

21) $f(x) = \cot 17x$

$f'(x) = -\csc^2 17x \cdot 17$

$= \boxed{-17\csc^2 17x}$

22) $f(x) = (\sin 5x)^4$

$f'(x) = 4(\sin 5x)^3(\cos 5x)5$

$= \boxed{20(\sin 5x)^3(\cos 5x)}$

23) $f(x) = x^2\sqrt{x^2 - 9}$

$f'(x) = x^2 \cancel{\tfrac{1}{2}}(x^2-9)^{-1/2}(\cancel{2}x) + \sqrt{x^2-9} \cdot 2x$

$f'(x) = x\left(\frac{x^2}{\sqrt{x^2-9}} + 2\sqrt{x^2-9}\right)$

24) $f(x) = 3x^3(5x^2+7)^5$

$f'(x) = 3x^3 \cdot 5(5x^2+7)^4(10x) + (5x^2+7)^5(9x^2)$

$3x^2(5x^2+7)^4[50x^2 + 3(5x^2+7)]$

$\boxed{3x^2(5x^2+7)^4[65x^2+21]}$

25) $f(x) = \frac{x^2+5}{x^3-9}$

$$f'(x) = \frac{(x^3-9)(2x) - (x^2+5)(3x^2)}{(x^3-9)^2}$$

$$\frac{x[2(x^3-9) - 3x(x^2+5)]}{(x^3-9)^2}$$

$$\frac{x[2x^3 - 18 - 3x^3 - 15x]}{(x^3-9)^2}$$

$$\boxed{\frac{x[-x^3 - 15x - 18]}{(x^3-9)^2}}$$