AP Calculus Review

KEY

① $y = x^5 + 5x^4 - 10x^2 + 6$

$\boxed{y' = 5x^4 + 20x^3 - 20x}$

② $y = 3x^{1/2} - x^{3/2} + 2x^{-1/2}$

$y' = \frac{3}{2}x^{-1/2} - \frac{3}{2}x^{1/2} - x^{-3/2}$

$\boxed{y' = \frac{3}{2\sqrt{x}} - \frac{3\sqrt{x}}{2} - \frac{1}{x\sqrt{x}}}$

③ $y = \frac{1}{2}x^{-2} + 4x^{-1/2}$

$y' = -x^{-3} - 2x^{-3/2}$

$\boxed{y' = \frac{-1}{x^3} - \frac{2}{x\sqrt{x}}}$ ← $x^{3/2} = \sqrt{x^3} = x\sqrt{x}$

④ $y = \sqrt{2x} + 2\sqrt{x}$

$y = (2x)^{1/2} + 2x^{1/2}$

$y' = \frac{1}{2}(2x)^{-1/2}(2) + x^{-1/2}$

$\boxed{y' = \frac{1}{\sqrt{2x}} + \frac{1}{\sqrt{x}}}$

⑤ $f(t) = \frac{2}{\sqrt{t}} + \frac{6}{\sqrt[3]{t}}$

$f(t) = 2t^{-1/2} + 6t^{-1/3}$

$f'(t) = -t^{-3/2} - 2t^{-4/3}$

$\boxed{f'(t) = \frac{-1}{t\sqrt{t}} - \frac{2}{t\sqrt[3]{t}}}$ ← $t^{4/3} = \sqrt[3]{t^4} = t\sqrt[3]{t}$

⑥ $y = (1-5x)^6$

$y' = 6(1-5x)^5(-5)$

$\boxed{y' = -30(1-5x)^5}$

⑦ $f(x) = (3x - x^3 + 1)^4$

$f'(x) = 4(3x - x^3 + 1)^3(3 - 3x^2)$

$\boxed{f'(x) = 12(1-x^2)(3x - x^3 + 1)^3}$

⑧ $y = (3+4x-x^2)^{1/2}$

$y' = \frac{1}{2}(3+4x-x^2)^{-1/2}(4-2x)$

$y' = (2-x)(3+4x-x^2)^{-1/2}$

$$\boxed{y' = \frac{2-x}{\sqrt{3+4x-x^2}}}$$

⑨ $\theta = \frac{3r+2}{2r+3}$

$$\theta' = \frac{(2r+3)(3)-(3r+2)(2)}{(2r+3)^2}$$

$$\theta' = \frac{6r+9-6r-4}{(2r+3)^2} = \boxed{\frac{5}{(2r+3)^2}}$$

⑩ $y = \left(\frac{x}{1+x}\right)^5 = \frac{x^5}{(1+x)^5}$

$$y' = \frac{(1+x)^5(5x^4)-(x^5)(5)(1+x)^4}{[(1+x)^5]^2}$$

$$y' = \frac{5x^4(1+x)^4[(1+x)-x]}{(1+x)^{10}}$$

$$y' = \frac{5x^4(1+x)^4}{(1+x)^{10}} = \boxed{\frac{5x^4}{(1+x)^6}}$$

⑪ $y = 2x^2\sqrt{2-x} = 2x^2(2-x)^{1/2}$

$y' = (2x^2)(\frac{1}{2})(2-x)^{-1/2}(-1) + (2-x)^{1/2}(4x)$

$$= \frac{-x^2+4x(2-x)}{\sqrt{2-x}} = \frac{-x^2+8x-4x^2}{\sqrt{2-x}}$$

$$\boxed{\frac{-5x^2+8x}{\sqrt{2-x}}}$$

⑫ $f(x) = x\sqrt{3-2x^2} = x(3-2x^2)^{1/2}$

$f'(x) = x(\frac{1}{2})(3-2x^2)^{-1/2}(-4x) + (3-2x^2)^{1/2}$

$$f'(x) = \frac{-2x^2}{\sqrt{3-2x^2}} + \sqrt{3-2x^2}$$

$$f'(x) = \frac{-2x^2+(3-2x^2)}{\sqrt{3-2x^2}} = \boxed{\frac{3-4x^2}{\sqrt{3-2x^2}}}$$

(13)
$$y = (x-1)\sqrt{x^2-2x+2}$$
$$y = (x-1)(x^2-2x+2)^{1/2}$$
$$y' = (x-1)\left(\tfrac{1}{2}\right)(x^2-2x+2)^{-1/2}(2x-2) + (x^2-2x+2)^{1/2}$$
$$y' = \frac{(x-1)(x-1)}{\sqrt{x^2-2x+2}} + \sqrt{x^2-2x+2}$$
$$y' = \frac{x^2-2x+1 + x^2-2x+2}{\sqrt{x^2-2x+2}} = \boxed{\frac{2x^2-4x+3}{\sqrt{x^2-2x+2}}}$$

(14)
$$z = \frac{w}{\sqrt{1-4w^2}} = w(1-4w^2)^{-1/2}$$
$$z' = w\left(-\tfrac{1}{2}\right)(1-4w^2)^{-3/2}(-8w) + (1-4w^2)^{-1/2}$$
$$z' = \frac{4w^2}{(1-4w^2)^{3/2}} + \frac{1}{(1-4w^2)^{1/2}} = \frac{4w^2 + (1-4w^2)}{(1-4w^2)^{3/2}}$$
$$\boxed{z' = \frac{1}{(1-4w^2)^{3/2}}}$$

(15)
$$y = \sqrt{1+\sqrt{x}} = (1+x^{1/2})^{1/2}$$
$$y' = \tfrac{1}{2}(1+x^{1/2})^{-1/2}\left(\tfrac{1}{2}x^{-1/2}\right)$$
$$\left(\frac{1}{2\sqrt{1+x^{1/2}}}\right)\left(\frac{1}{2\sqrt{x}}\right) = \boxed{\frac{1}{4\sqrt{x+x^{3/2}}}}$$

(16)
$$f(x) = \sqrt{\frac{x-1}{x+1}} = \frac{\sqrt{x-1}}{\sqrt{x+1}} = \frac{(x-1)^{1/2}}{(x+1)^{1/2}}$$
$$f'(x) = \frac{(x+1)^{1/2}\left(\tfrac{1}{2}\right)(x-1)^{-1/2} - (x-1)^{1/2}\left(\tfrac{1}{2}\right)(x+1)^{-1/2}}{\left[(x+1)^{1/2}\right]^2}$$
$$f'(x) = \frac{\dfrac{\sqrt{x+1}}{2\sqrt{x-1}} - \dfrac{\sqrt{x-1}}{2\sqrt{x+1}}}{x+1} = \frac{x+1-x+1}{2\sqrt{x-1}\sqrt{x+1}} = \frac{2}{2\sqrt{x-1}\sqrt{x+1}} \cdot \frac{1}{x+1} = \boxed{\frac{2}{2\sqrt{x-1}(x+1)^{3/2}}}$$

(17)
$$y = (x^2+3)^4(2x^3-5)^3$$
$$(x^2+3)^4(3)(2x^3-5)^2(6x^2) + (2x^3-5)^3(4)(x^2+3)^3(2x)$$
$$\left[(x^2+3)^3(2x^3-5)^2(2x)\right]\left[9(x^2+3)x + (2x^3-5)(4)\right]$$
$$\left[(x^2+3)^3(2x^3-5)^2(2x)\right]\left[9x^3+27x+8x^3-20\right]$$
$$\left[(x^2+3)^3(2x^3-5)^2(2x)\right]\left[17x^3+27x-20\right]$$

(18) $S = \frac{t^2+2}{3-t^2}$

$S' = \frac{(3-t^2)(2t)-(t^2+2)(-2t)}{(3-t^2)^2} = \frac{6t-2t^3+2t^3+4t}{(3-t^2)^2} = \boxed{\frac{10t}{(3-t^2)^2}}$

(19) $y = \left(\frac{x^3-1}{2x^3+1}\right)^4 = \frac{(x^3-1)^4}{(2x^3+1)^4}$ $y' = \frac{(2x^3+1)^4(4)(x^3-1)^3(3x^2)-(x^3-1)^4(4)(2x^3+1)^3(6x^2)}{(2x^3+1)^8}$

$y' = (2x^3+1)^3(x^3-1)^3(12x^2)[(2x^3+1)-(x^3-1)(2)]$

$= \boxed{(2x^3+1)^3(x^3-1)^3(12x^2)(3)}$

(20) $y = e^{5x}$

$\boxed{y' = 5e^{5x}}$

(21) $y = \ln\sqrt{3-x^2}$

$y = \ln(3-x^2)^{1/2} = \frac{1}{2}\ln(3-x^2)$

$y' = \frac{1}{2}\left(\frac{-2x}{3-x^2}\right) = \boxed{\frac{-x}{3-x^2}}$

(22) $y = \ln 3x^5 = \ln 3 + \ln x^5$

$= \ln 3 + 5\ln x$

$\boxed{y' = \frac{5}{x}}$

(23) $y = \ln(x^2+x-1)^3$

$y = 3\ln(x^2+x-1)$

$\boxed{y' = 3\left(\frac{2x+1}{x^2+x-1}\right)}$

(24) $y = x \cdot \ln x - x$

$y' = x\left(\frac{1}{x}\right) + \ln x - 1$

$\boxed{y' = \ln x}$

(27) $y = x^{\ln x}$

$\ln y = \ln x \ln x$

$\frac{y'}{y} = \ln x\left(\frac{1}{x}\right) + \ln x\left(\frac{1}{x}\right)$

$\frac{y'}{y} = 2\left[\ln x\left(\frac{1}{x}\right)\right]$

$\boxed{y' = 2\left[\ln x\left(\frac{1}{x}\right)\right]x^{\ln x}}$

(28) $y = x^{e^{-x^2}}$

$\ln y = \ln x^{e^{-x^2}}$

$\ln y = e^{-x^2}\ln x$

$\frac{y'}{y} = e^{-x^2}\left(\frac{1}{x}\right) + \ln x\left(e^{-x^2}\right)(-2x)$

$\frac{y'}{y} = e^{-x^2}\left[\frac{1}{x} + \ln x(-2x)\right]$

$\boxed{y' = e^{-x^2}\left[\frac{1}{x} - 2x\ln x\right]\left[x^{e^{-x^2}}\right]}$

y'''

(25) $y = x^2e^x$

$y' = x^2e^x + e^x 2x = e^x(x^2+2x)$

$y'' = e^x(2x+2) + (x^2+2x)e^x$

$= e^x(2x+2+x^2+2x)$

$= e^x(4x+x^2+2)$

$y''' = e^x(4+2x) + (4x+x^2+2)e^x$

$= e^x(4+2x+4x+x^2+2) =$

$\boxed{e^x(x^2+6x+6)}$

(26) $y = x^x$

$\ln y = x\ln x \rightarrow \frac{y'}{y} = x\left(\frac{1}{x}\right) + \ln x \rightarrow \boxed{y' = (1+\ln x)x^x}$

$\frac{y'}{y} = 1 + \ln x$