Name_____ Date_____ Period____

—The Fundamental Theorem of Calculus

Show all work on a separate sheet of paper. No calculator unless otherwise stated.

Multiple Choice

1. (Calculator Permitted) What is the average value of $f(x)=\cos x$ on the interval $[1,5]$?

   (A) –0.990 (B) –0.450 (C) –0.128 (D) 0.412 (E) 0.998

2. If the average value of the function f on the interval $[a,b]$ is 10, then $\int_a^b f(x)dx =$

   (A) $\frac{10}{b-a}$ (B) $\frac{f(a)+f(b)}{10}$ (C) $10b-10a$ (D) $\frac{b-a}{10}$ (E) $\frac{f(a)+f(b)}{20}$

3. (Calculator Permitted) Let $f'(x)=\ln(2+\sin x)$. If $f(3)=4$, then $f(5)=$

   (A) 0.040 (B) 0.272 (C) 0.961 (D) 4.555 (E) 6.667

4. What is $\lim_{h\to 0}\frac{1}{h}\int_x^{x+h} f(t)dt$?

   (A) 0 (B) 1 (C) $f'(x)$ (D) $f(x)$ (E) nonexistent

5. What is the linearization of $f(x)=\int_\pi^x \cos^3 t\,dt$ at $x=\pi$?

   (A) $y=-1$ (B) $y=-x$ (C) $y=\pi$ (D) $y=x-\pi$ (E) $y=\pi-x$

6. (Calculator Permitted) The area of the region enclosed between the graph of $y=\sqrt{1-x^4}$ and the x-axis is

   (A) 0.886 (B) 1.253 (C) 1.414 (D) 1.571 (E) 1.748

Short Answer

7. Let f be a function such that $f''(x)=6x+12$.

   a) Find $f(x)$ if the graph of f is tangent to the line $4x-y=5$ at the point $(0,-5)$

   b) Find the average value of $f(x)$ on the closed interval $[-1,1]$.

8. Suppose f has a negative derivative for all values of x and that $f(1)=0$. Which of the following statements must be true of the function

$$h(x)=\int_0^x f(t)\,dt\ ?$$

Give reasons for your answers.

a) h is a twice-differentiable function of x.
b) h and dh/dx are both continuous.
c) The graph of h has a horizontal tangent at $x=1$.
d) h has a local maximum at $x=1$..
e) h has a local minimum at $x=1$.
f) The graph of h has an inflection point at $x=1$.
g) The graph of dh/dx crosses the x-axis at $x=1$.

9. Find $\dfrac{dy}{dx}$

a) $y=\int_{-\pi}^{x}\dfrac{2-\sin t}{3+\cos t}\,dt$ b) $y=\int_{x}^{7}\sqrt{2m^4+m+1}\,dm$ c) $y=\int_{x^3}^{5}\dfrac{\cos t}{t^2+1}\,dt$ d) $y=\int_{\sqrt{x}}^{x^3}\sqrt{u}\sin u\,du$

10. If $F(x)=\int_1^x f(t)\,dt$, where $f(t)=\int_1^{t^2}\dfrac{\sqrt{1+u^4}}{u}\,du$, find $F''(2)$.

11. (Calculator Active) If $\dfrac{dy}{dx}=\sin^3 x$ and $y=4$ when $x=5$, construct and evaluate an integral equation to find (a) $y(7)$ (b) $y(0)$ (c) $y(-2)$ (d) $y(x)$

12. Evaluate without a calculator, then verify using fnINT(

a) $\int_2^{-1}3^x\,dx$ b) $\int_{-2}^{-1}\dfrac{1}{x^2}\,dx$ c) $\int_0^1\left(x^2+\sqrt{x}\right)dx$ d) $\int_{\pi/6}^{5\pi/6}\csc^2\theta\,d\theta$ e) $\int_0^4\dfrac{1-\sqrt{u}}{\sqrt{u}}\,du$

f) $\int_0^2 x\left(2+x^5\right)dx$ g) $\int_0^1\dfrac{4}{t^2+1}\,dt$ h) $\int_0^2 f(x)\,dx$ where $f(x)=\begin{cases}x^4, & 0\le x<1\\ x^5, & 1\le x\le 2\end{cases}$

13. Find the area of the region bounded by the x-axis and the curve $y=x^3-4x$ on $-2\le x\le 2$

14. If $f(1)=12$, $f'(x)$ is continuous, and $\int_1^4 f'(x)=17$, what is the value of $f(4)$?

15. Find the average value of the following function on the given intervals. Verify with fnINT(

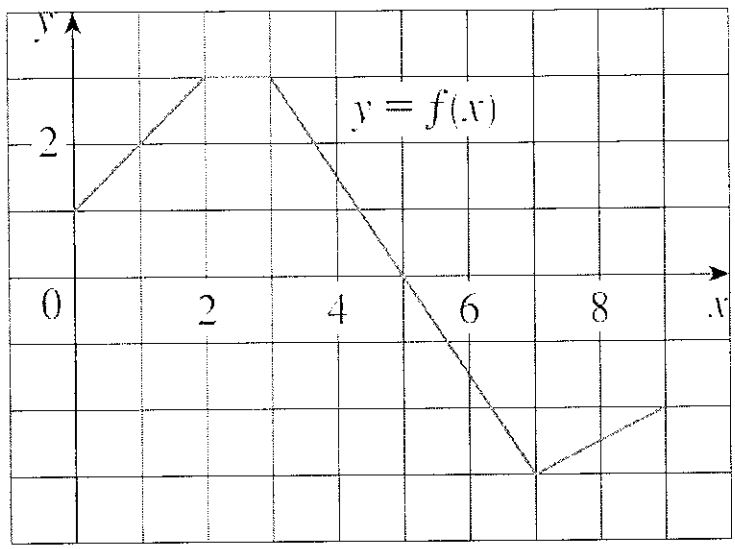
a) $f(x)=\cos x$ on $[0,\pi/2]$ b) $f(x)=1/x$ on $[1,4]$ c) $y=\sec x\tan x$ on $[0,\pi/4]$

16. The graph of f is shown. If $F(x)=\int_2^x f(t)\,dt$, evaluate the following using areas to help you.

a) $F(0)$ b) $F(2)$ c) $F(5)$ d) $F(7)-F(5)$ e) $F(9)$

f) where does F have a maximum value? A minimum value?

g) What is the average value of $f(x)$ on $[2,9]$?



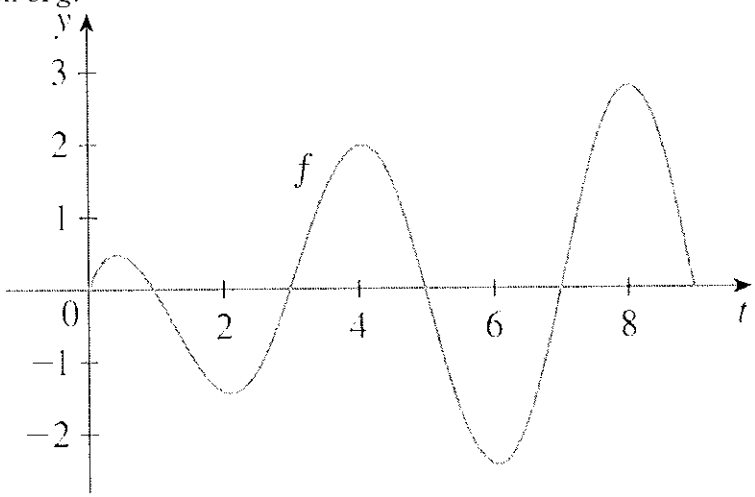
17. Let $g(x)=\int_0^x f(t)\,dt$, where f is the function whose graph is given below.

a) At what values of x do the local maximum and local minimum of g occur?

b) Where does g attain its absolute maximum value?

c) On what approximate intervals is g concave downward?

d) Sketch the graph of g.



18. (Calculator Permitted) If a cup of coffee has temperature $95^\circ C$ in a room where the temperature is $20^\circ C$, then, according to Newton's Law of Cooling, the temperature of the coffee after t minutes is $T(t)=20+75e^{-t/50}$. What is the average temperature of the coffee during the first half hour? Show your integral set up. Include units in your final answer.