

## 2. Integrals of Trigonometric

### 2.9 Integration and Completing the Square of Inverse Trig Functions

Evaluate  $\int 5 \sec x \tan x \, dx$ .

- a)  $5 \sec^3 x \tan x + C$                       (b)  $5 \sec x + C$   
c)  $\frac{1}{5} \sec^3 x \tan x + C$                       (d)  $5[\sec^3 x + \sec x \tan^2 x] + C$   
e) None of these

Evaluate  $\int \frac{\sin^3 \theta}{1 - \cos^2 \theta} \, d\theta$ .

- a)  $-\cos \theta + C$                       (b)  $\cos \theta + C$   
c)  $\frac{\cos \theta [3 - 3 \cos^2 \theta - 2 \sin^2 \theta]}{1 - \cos^2 \theta}$                       (d)  $\frac{1}{2} \sin^2 \theta + C$   
e) None of these

Evaluate  $\int 3 \csc^2 x \, dx$ .

- (a)  $\frac{1}{3} \csc^3 x + C$                       (b)  $6 \csc^2 x \cot x + C$                       (c)  $-3 \cot x + C$   
(d)  $-\frac{1}{3} \csc^3 x + C$                       (e) None of these

Evaluate  $\int \frac{\sec^3 \theta \tan \theta}{1 + \tan^2 \theta} \, d\theta$ .

- (a)  $\frac{1}{4} \sec^4 \theta + C$                       (b)  $\frac{1}{2} \sec^2 \theta + C$                       (c)  $\frac{1}{4} \sec^2 \theta \tan^2 \theta + C$   
(d)  $\sec \theta + C$                       (e) None of these

Evaluate  $\int 3 \csc x \cot x \, dx$ .

Evaluate  $\int \frac{\cos^3 \theta}{2 - 2 \sin^2 \theta} \, d\theta$ .

Evaluate  $\int_{\pi/4}^{\pi/3} \sec^2 x \, dx$ .

Find the average value of  $f(x) = \sin x$  on the interval  $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ .

9. Evaluate  $\int \cos 3x \, dx$ .

- (a)  $\sin 3x + C$                       (b)  $-\sin 3x + C$                       (c)  $-\sin \frac{3}{2} x^2 + C$   
(d)  $\frac{1}{3} \sin 3x + C$                       (e) None of these

10. Evaluate  $\int \sin^3 3x \cos 3x \, dx$ .

- (a)  $\frac{1}{8} \sin^4 3x \cos^2 3x + C$                       (b)  $\frac{1}{4} \sin^4 3x + C$   
(c)  $3 \sin^2 3x (3 \cos^2 3x - \sin^2 3x) + C$                       (d)  $\frac{1}{12} \sin^4 3x + C$   
(e) None of these

11. Evaluate  $\int \sin \frac{x}{2} \, dx$ .

- (a)  $\cos \frac{x}{2} + C$                       (b)  $-2 \cos \frac{x}{2} + C$                       (c)  $\sin \frac{x^2}{4} + C$   
(d)  $2 \sin^2 \frac{x}{2} + C$                       (e) None of these

12. Evaluate  $\int \frac{\sec^2 x}{\sqrt{\tan x}} \, dx$ .

13. Evaluate  $\int \tan 3x \, dx$ .

- (a)  $\frac{1}{3} \ln |\sec 3x| + C$                       (b)  $3 \sec^2 3x + C$                       (c)  $\frac{1}{3} \sec^2 3x$   
(d)  $\ln |\cos 3x| + C$                       (e) None of these

14. Evaluate  $\int \frac{\sin^2 x - \cos^2 x}{\sin x} \, dx$ .

- (a)  $-2 \cos x + \ln |\csc x + \cot x| + C$                       (b)  $-\ln |\csc x + \cot x| + C$   
(c)  $-\sec x + C$                       (d)  $\cos x + \ln |\csc x + \cot x| + C$   
(e) None of these

15. Evaluate  $\int x \cot x^2 \, dx$ .

- (a)  $\frac{x^2}{2} \sec^2 x^2 + C$                       (b)  $\frac{x^2}{4} \ln |\sin x^2| + C$                       (c)  $x \cot x^2 \csc x^2 + C$   
(d)  $\frac{1}{2} \ln |\sin x^2| + C$                       (e) None of these

16. Evaluate  $\int \frac{\cos^3 x - \sin^2 x}{\cos^2 x} \, dx$ .

- (a)  $\frac{\cos^2 x}{2} - \tan x + x + C$                       (b)  $\sin x - \sec x + C$   
(c)  $\sin x - \tan x + x + C$                       (d)  $\sin x - \frac{\tan^3 x}{3} + C$   
(e) None of these

7. Evaluate  $\int \sec 2x \, dx$ .

8. Find the area of the region bounded by the graphs of  $f(x) = \sin x$  and  $g(x) = \cos x$ , for  $\pi/4 \leq x \leq 5\pi/4$ .

9. Find the volume of the solid formed by revolving the region bounded by  $y = \sin x$  and  $y = 0$  in the interval  $[0, \pi]$  about the  $y$  axis.

- (a)  $\pi^3$  (b)  $\frac{1}{2} \pi^2$  (c)  $2 \pi^2$   
(d)  $\pi$  (e) None of these

10. Write the definite integral that represents the arc length of one period of the curve  $y = \sin 2x$ . (Do not evaluate the integral.)

11. Evaluate  $\int \frac{x+3}{x^2+9} \, dx$ .

- (a)  $\ln|x-3| + C$  (b)  $\frac{1}{3} \arctan \frac{x}{3} + C$   
(c)  $\frac{1}{2} \ln(x^2+9) + \arctan \frac{x}{3} + C$  (d)  $\ln(x^2+9) + \frac{1}{3} \arctan \frac{x}{3} + C$   
(e) None of these

12. Evaluate  $\int \frac{dx}{\sqrt{8+2x-x^2}}$ .

- (a)  $\ln \sqrt{8+2x-x^2}$  (b)  $\arcsin \frac{x-1}{3} + C$  (c)  $\sqrt{8+2x-x^2} + C$   
(d)  $\frac{1}{3} \operatorname{arcsec} \frac{x-1}{3} + C$  (e) None of these

13. Evaluate  $\int \frac{x+2}{\sqrt{4-x^2}} \, dx$ .

- (a)  $-\frac{1}{2} \sqrt{4-x^2} + 2 \arcsin \frac{x}{2} + C$  (b)  $-\sqrt{4-x^2} + 2 \arcsin \frac{x}{2} + C$   
(c)  $\ln|2-x| + C$  (d)  $x^2 + 2x + \arcsin \frac{x}{2} + C$   
(e) None of these

14. Evaluate  $\int \frac{5}{x^2+6x+13} \, dx$ .

- (a)  $5 \ln|x^2+6x+13| + C$  (b)  $5 \left( \frac{x^3}{3} + 3x^2 + 13x \right) + C$   
(c)  $\frac{5}{2} \arctan \frac{x+3}{2} + C$  (d)  $-\frac{5}{x} + \frac{5}{6} \ln|x| + \frac{5}{13}x + C$

Find the indefinite integral:  $\int \frac{x}{16+x^4} dx$ .

(a)  $\frac{1}{2} \arcsin \frac{x^2}{4} + C$

(b)  $\frac{1}{8} \arctan \frac{x^2}{4} + C$

(c)  $\frac{1}{4} \arctan \frac{x^2}{4} + C$

(d)  $\frac{1}{8} \operatorname{arcsec} \frac{x^2}{4} + C$

(e) None of these

Find the indefinite integral:  $\int \frac{x}{81+x^4} dx$ .

(a)  $\frac{1}{2} \arcsin \frac{x^2}{9} + C$

(b)  $\frac{1}{18} \arctan \frac{x^2}{9} + C$

(c)  $\frac{1}{18} \operatorname{arcsec} \frac{x^2}{9} + C$

(d)  $\frac{1}{9} \arctan \frac{x^2}{9} + C$

(e) None of these

Find the indefinite integral:  $\int \frac{x}{36+x^4} dx$ .

(a)  $\frac{1}{12} \arctan \frac{x^2}{6} + C$

(b)  $\frac{1}{2} \arcsin \frac{x^2}{6} + C$

(c)  $\frac{1}{6} \arctan \frac{x^2}{6} + C$

(d)  $\frac{1}{12} \operatorname{arcsec} \frac{x^2}{6} + C$

(e) None of these

Evaluate  $\int \frac{5x+16}{x^2+9} dx$ .

5 . . . . 16 . . . . x . . .

Evaluate  $\int \frac{1}{\sqrt{-3+4x-x^2}} dx$ .