Name: Key

AP® Calculus AB
Answer Sheet
for Multiple-Choice Section

No.	Answer
1	B
2	D
3	B
4	B
5	C
6	A
7	B
8	D
9	A
10	D
11	C
12	D
13	A
14	D
15	B
16	D
17	A
18	D
19	B
20	C
21	B
22	C
23	C
24	C
25	B
26	A
27	A
28	C
29	C
30	D

No.	Answer
76	A
77	B
78	D
79	B
80	C
81	A
82	A
83	C
84	D
85	D
86	B
87	D
88	C
89	C
90	B

Question 1

(a) $h'(6) \approx \dfrac{h(7) - h(5)}{7 - 5} = \dfrac{18.5 - 15.5}{2} = 1.5$ in/min

1 : answer with units

(b) $\int_0^{10} h(t)\,dt \approx (2-0)\cdot h(2) + (5-2)\cdot h(5) + (7-5)\cdot h(7) + (10-7)\cdot h(10)$

$= 2(10.0) + 3(15.5) + 2(18.5) + 3(20.0) = 163.5$

Because h is an increasing function, the right Riemann sum approximation is greater than $\int_0^{10} h(t)\,dt$.

3 :
- 1 : right Riemann sum
- 1 : approximation
- 1 : overestimate with reason

(c) Average depth in tank $B = \dfrac{1}{10}\int_0^{10} g(t)\,dt = 16.624$ in

Average depth in tank $A = \dfrac{1}{10}\int_0^{10} h(t)\,dt < \dfrac{1}{10}(163.5) = 16.35 \text{ in} < 16.624$ in

Therefore, the average depth of the water in tank B is greater than the average depth of the water in tank A.

3 :
- 1 : integral
- 1 : average depth in tank B
- 1 : answer with reason

(d) $g'(6) = 0.887$

The depth of the water in tank B is increasing at time $t = 6$ because $g'(6) > 0$.

2 :
- 1 : uses $g'(6)$
- 1 : answer with reason

$\int \quad dx$

Question 2

(a) $a_Q(t) > 0$ for $0 < t < 3.963$.

The velocity of particle Q is increasing on the interval $(0, 3.963]$.

$2: \begin{cases} 1: \text{interval} \\ 1: \text{reason} \end{cases}$

(b) $x_Q(3) = x_Q(0) + \int_0^3 v_Q(t)\,dt$

$= 2 + (-3.490102)$

$= -1.490$

$2 + \int_0^3 v_Q(t)\,dt$

$3: \begin{cases} 1: \text{integral} \\ 1: \text{uses } x_Q(0) \\ 1: \text{answer} \end{cases}$

(c) $x_R(t)$ is differentiable. $\Rightarrow$ $x_R(t)$ is continuous on $1 \le t \le 3$.

$$\frac{x_R(3) - x_R(1)}{3 - 1} = \frac{8 - 4}{2} = 2$$

Therefore, by the Mean Value Theorem, there is at least one time t, $1 < t < 3$, for which $x_R'(t) = v_R(t) = 2$.

$2: \begin{cases} 1: \dfrac{x_R(3) - x_R(1)}{3 - 1} \\ 1: \text{conclusion, using MVT} \end{cases}$

(d) $x_Q(3) = -1.490$ and $v_Q(3) = 1.682$ (or 1.681)
Particle Q is moving to the right from $x = -1.490$.

$x_R(3) = 8$ and $v_R(3) = -2$
Particle R is moving to the left from $x = 8$.

Therefore, the particles are moving toward each other.

$2: \begin{cases} 1: \text{answer} \\ 1: \text{explanation} \end{cases}$

Question 3

(a) $f'(x) = g(x)$ The function f has critical points at $x = -6$ and $x = 2$. f has a local minimum at $x = -6$ because f' changes from negative to positive at that value. f has a local maximum at $x = 2$ because f' changes from positive to negative at that value.	$3 : \begin{cases} 1 : f'(x) = g(x) \\ 1 : \text{critical points} \\ 1 : \text{classifications with justification} \end{cases}$
(b) $f(0) = \int_{-8}^{0} g(t)\,dt = 8 + 4\pi$	1 : answer
(c) $\lim_{x \to -4} f(x) = f(-4) = \int_{-8}^{-4} g(t)\,dt = 0$ $\lim_{x \to -4} \left(x^2 + 4x\right) = 0$ Using L'Hospital's Rule, $\lim_{x \to -4} \frac{f(x)}{x^2 + 4x} = \lim_{x \to -4} \frac{f'(x)}{2x + 4} = \lim_{x \to -4} \frac{g(x)}{2x + 4} = \frac{2}{-4} = -\frac{1}{2}$	$2 : \begin{cases} 1 : \text{L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$
(d) $h'(x) = \frac{\left(x^2 + 1\right)g'(x) - g(x)2x}{\left(x^2 + 1\right)^2}$ $h'(1) = \frac{\left(1^2 + 1\right)g'(1) - g(1)(2)(1)}{\left(1^2 + 1\right)^2}$ $= \frac{2(-3) - 3(2)}{4} = -3$	$3 : \begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$

Question 4

(a) $\frac{dy}{dx} = (y-2)(x^2+1)$

$\int \frac{dy}{y-2} = \int (x^2+1)\,dx$

$\ln|y-2| = \frac{x^3}{3} + x + C$

$\ln 3 = \frac{0^3}{3} + 0 + C \Rightarrow C = \ln 3$

Because $y(0) = 5$, $y > 2$, so $|y-2| = y-2$.

$y - 2 = 3e^{\frac{x^3}{3}+x}$

$y = 2 + 3e^{\frac{x^3}{3}+x}$

Note: this solution is valid for all real numbers.

5 :
- 1 : separation of variables
- 2 : antiderivatives
- 1 : constant of integration and uses initial condition
- 1 : solves for y

Note: max 3/5 [1-2-0-0] if no constant of integration

Note: 0/5 if no separation of variables

(b) $\lim_{x\to-\infty}\left(2 + 3e^{\frac{x^3}{3}+x}\right) = 2$

1 : answer

(c) $\left.\frac{dy}{dx}\right|_{(1,3)} = (3-2)(1^2+1) = 2$

$\frac{d^2y}{dx^2} = \frac{dy}{dx}(x^2+1) + (y-2)(2x)$

$\left.\frac{d^2y}{dx^2}\right|_{(1,3)} = (2)(1^2+1) + (3-2)(2) = 6$

Because $\left.\frac{d^2y}{dx^2}\right|_{(1,3)} > 0$ and $\frac{d^2y}{dx^2}$ is continuous, the graph of $y = f(x)$ is concave up at the point $(1, 3)$.

3 :
- 2 : $\left.\frac{d^2y}{dx^2}\right|_{(1,3)}$
- 1 : concave up with reason

Question 5

(a) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \left(3x^2 + 2x\right) = 0$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \left(e^{2x} + 2\right) = 3$

Therefore, $\lim_{x\to 0} f(x)$ does not exist, and f is not continuous at $x = 0$.

$2 : \begin{cases} 1 : \text{considers one-sided limits} \\ 1 : \text{answer with justification} \end{cases}$

(b) $f'(-2) = (6x + 2)\big|_{x=-2} = 6(-2) + 2 = -10$

$f'(3) = 2e^{2x}\Big|_{x=3} = 2e^{(2)(3)} = 2e^6$

$2 : \begin{cases} 1 : f'(-2) \\ 1 : f'(3) \end{cases}$

(c) $f'(0)$ does not exist because f is not continuous at $x = 0$.

$1 : \text{explanation}$

(d) $g(1) = \int_{-1}^{1} f(t)\, dt = \int_{-1}^{0} \left(3t^2 + 2t\right) dt + \int_{0}^{1} \left(e^{2t} + 2\right) dt$

$= \left[t^3 + t^2\right]_{-1}^{0} + \left[\frac{e^{2t}}{2} + 2t\right]_{0}^{1}$

$= [(0 + 0) - (-1 + 1)] + \left[\left(\frac{e^2}{2} + 2\right) - \left(\frac{1}{2} + 0\right)\right]$

$= \frac{e^2}{2} + \frac{3}{2}$

$4 : \begin{cases} 1 : \text{integrals} \\ 2 : \text{antiderivatives} \\ 1 : \text{answer} \end{cases}$

Question 6

(a) $\int_0^2 L(t)\,dt = \int_0^2 (-2t + 15)\,dt = \left[-t^2 + 15t\right]_0^2$
$= -4 + 30 = 26$

26 hundred bees leave the hive during the time interval $0 \le t \le 2$.

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) The total number of bees, in hundreds, in the hive at time t is
$35 + \int_0^t (E(x) - L(x))\,dx$.

$$\begin{aligned} 35 + \int_0^4 (E(x) - L(x))\,dx &= 35 + \int_0^4 \left(\left(16x - 3x^2\right) - (-2x + 15)\right) dx \\ &= 35 + \int_0^4 \left(-3x^2 + 18x - 15\right) dx \\ &= 35 + \left[-x^3 + 9x^2 - 15x\right]_0^4 \\ &= 35 + (-64 + 144 - 60) \\ &= 55 \end{aligned}$$

55 hundred bees are in the hive at time $t = 4$.

$3 : \begin{cases} 1 : \text{expression for total} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) Let $B(t)$ be the total number of bees, in hundreds, in the hive at time t, for $0 \le t \le 4$.

$B(t) = 35 + \int_0^t (E(x) - L(x))\,dx$

$B'(t) = E(t) - L(t) = -3t^2 + 18t - 15 = -3(t - 1)(t - 5)$

$B'(t) = 0 \Rightarrow t = 1, t = 5$

t	$B(t)$
0	35
1	28
4	55

The minimum numbers of bees in the hive for $0 \le t \le 4$ is 28 hundred bees.

$3 : \begin{cases} 1 : \text{sets } E(t) - L(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$