Venture Publishing 2006

Exam I section 1 Part A
no calculator

1) $f'(x) = \ln(x-2)$

$e^{\ln(x-2)} = e^0$

$x-2=1 \quad x=3$

$\longleftarrow - \ | \ + \longrightarrow$ (3)

$\boxed{2<x<3}$ A

2) $y = x\cos x$

$y' = -x\sin x + \cos x = 0$

$-x\sin x = -\cos x$

$x\sin x = \cos x$

$\boxed{\tan x = \frac{1}{x}}$ B

3) $F(x) = G[x + G(x)]$

$F'(x) = G'[x+G(x)](1+G'(x))$

$F'(1) = G'[1+G(1)](1+G'(1))$

$= G'(1+3)(1+-2)$

$= G'(4) \quad (-1)$

$= 2/3(-1) = \boxed{-2/3}$ E

4) $\int_2^6 \left(\frac{1}{x} + 2x\right)dx$

$(\ln|x| + x^2)\Big|_2^6$

$(\ln 6 + 36) - (\ln|2| + 4)$

$\ln 6 + 36 - \ln 2 - 4$

$\ln 6 - \ln 2 + 32 = \boxed{\ln 3 + 32}$ C

5) $f(x) = \frac{(\ln x)^2}{x}$

$f'(x) = \frac{x\, 2(\ln x)\left(\frac{1}{x}\right) - (\ln x)^2(1)}{x^2}$

$f'(x) = \frac{2\ln x - (\ln x)^2}{x^2} = \frac{\ln x(2-\ln x)}{x^2}$ E

$\ln x = 0 \quad x = 1$

$2 - \ln x = 0 \quad -\ln x = -2 \quad \ln x = 2$

und $x=0$

$\longleftarrow \cancel{-} \ + \ - \longrightarrow$ $0 \quad 1 \quad e^2$

6) $\frac{b-a}{n} = \frac{3-(-1)}{4} = 1 \quad \cancel{-1}, 0, 1, 2, 3$ right-hand

$1(3+1+1+3) = \boxed{8}$ D

7) $y = x^3 + 3x^2 + 2$

$y' = 3x^2 + 6x \quad y'' = 6x + 6 = 0$

$x = -1$

$3-6$

poi $(-1, 4)$ $\quad m = -3$

$y - 4 = -3(x+1) \quad y - 4 = -3x - 3$

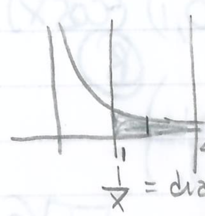
$\boxed{y = -3x + 1}$ A

8) $\int \cos(3-2x)\,dx \quad u = 3-2x$

$-\frac{1}{2}\int \cos u\, du \quad du = -2$

$\boxed{-\frac{1}{2}\sin(3-2x) + C}$ D

9) $\lim_{x\to\infty} \frac{\sqrt{9x^2+2}}{4x+3} = \boxed{3/4}$ B

10) 

$\frac{1}{x}$ = diameter

$\int_1^4 \frac{1}{2}\pi\left(\frac{1}{2}\cdot\frac{1}{x}\right)^2 dx$

$\frac{1}{8}\pi\int_1^4 \frac{1}{x^2}\,dx$

$\frac{1}{8}\pi\left(-\frac{1}{x}\right)\Big|_1^4$

B $\quad -\frac{\pi}{8x}\Big|_1^4 = -\frac{\pi}{32} + \frac{\pi}{8} = \boxed{\frac{3\pi}{32}}$

11) $f(x) = \ln x + e^{-x}$

$f'(x) = \frac{1}{x} - e^{-x}$

$f'(1) = 1 - e^{-1} = 1 - \frac{1}{e}$

$\longleftarrow - \ | \ + \longrightarrow$ 1 min

changes from dec/inc at 1.

A

$u =$

$du =$

12) $F(x) = \int_0^{x^2} \frac{1}{2+t^3}$

$F'(x) = \frac{1}{2+(x^2)^3} \cdot 2x$

$= \frac{2x}{2+x^6}$

$F'(-1) = \boxed{\frac{-2}{3}}$ E

13) $\frac{1}{1-(-1)} \int_{-1}^{1} 2t^3 - 3t^2 + 4$

$\frac{1}{2} \int_{-1}^{1} 2t^3 - 3t^2 + 4$

$\frac{1}{2}\left(\frac{2t^4}{4} - t^3 + 4t\right)\Big|_{-1}^{1}$

$\frac{1}{2}\left[\left(\frac{1}{2} - 1 + 4\right) - \left(\frac{1}{2} + 1 - 4\right)\right]$

$\frac{1}{2}\left[\frac{7}{2} + \frac{5}{2}\right] = \frac{1}{2}(6) = \boxed{3}$ C

14) Slopefield shows original which goes through (0,1) (cos x). $\frac{dy}{dx}$ must be sin x B

15) $\lim_{x \to 1} \frac{\sqrt{x} - 1}{(x-1)} \cdot \frac{(\sqrt{x}+1)}{(\sqrt{x}+1)}$

$= \frac{\cancel{x-1}}{\cancel{(x-1)}(\sqrt{x}+1)} = \frac{1}{\sqrt{x}+1} = \boxed{\frac{1}{2}}$ B

16) $y = \cos^2 x - \sin^2 x$ $(\cos x)^2$

$y' = 2\cos x(-\sin x) - 2\sin x \cos x$

$y' = -2\sin x \cos x - 2\sin x \cos x$

$\boxed{y' = -4\sin x \cos x}$ E

17) $\int_1^2 4x^3 + 6x - \frac{1}{x}\, dx$

$x^4 + 3x^2 - \ln|x| \Big|_1^2$

$(16 + 12 - \ln 2) - (1 + 3 - 0)$ C

$28 - \ln 2 - 4 = \boxed{24 - \ln 2}$

18) $\int \frac{x-2}{x-1} = \int \frac{x}{x-1} - \int \frac{2}{x-1}$

$u = x-1$, $du = 1$, $x = u+1$

$\int \frac{u+1}{u} \quad -2\ln|x-1| + C$

$\int 1 + \int \frac{1}{u} \quad -2\ln|x-1| + C$

$x + \ln|x-1| - 2\ln|x-1| + C$

$\boxed{x - \ln|x-1| + C}$ C

19) $g(-x) = g(x)$ sym to y

$g'(-x)(-1) = g'(x)$

$g'(-a)(-1) = g'(a)$

$g'(-a) = -g'(a)$ B

20) $y = \arctan \frac{1}{3}x$ $u = \frac{1}{3}x$ $(0,0)$ $\frac{u'}{1+u^2}$

$y' = \frac{1/3}{1+(\frac{1}{3}x)^2} = \frac{1/3}{1+\frac{1}{9}x^2}$

$y'(0) = 1/3$ (slope) $y = 1/3(x)$

$y = 1/3x$ $3y = x$

A $\boxed{x - 3y = 0}$

21) $\int_0^1 x^2 + 4 = \frac{x^3}{3} + 4x \Big|_0^1 = \frac{1}{3} + 4 = 4\frac{1}{3}$

$\int_1^3 6 - x = 6x - \frac{x^2}{2}\Big|_1^3 = (18 - \frac{9}{2}) - (6 - \frac{1}{2}) = \frac{27}{2} - \frac{11}{2} = 8$

$4\frac{1}{3} + 8 = 12\frac{1}{3}$ $\boxed{C}$

22) $\frac{d}{dx}(\ln e^{3x}) = \frac{d}{dx} 3x = \boxed{3}$ B

$\cancel{\frac{3e^{3x}}{e^{3x}}}$

23) $g'(x) = 2g(x)$ $g(-1) = 1$

If $g(x) = e^{2x}$ $g'(e^{2x}) = 2e^{2x} \neq 2e^{2x}$

If $g(x) = e^{-x}$ $g'(e^{-x}) = -e^{-x} \neq 2e^{-2x}$

If $g(x) = e^{x+1}$ $g'(e^{x+1}) = e^{x+1} \neq 2e^{x+1}$

If $g(x) = e^{2x+2}$ $g'(e^{2x+2}) = 2e^{2x+2} = 2e^{2x+2}$ $g(-1) = 1$ ✓ D

If $g(x) = e^{2x-2}$ $g'(e^{2x-2}) = 2e^{2x-2} = 2e^{2x-2}$ $g(-1) \neq 1$

24) $a(t) = 3t+2$

$v(t) = \int 3t+2 = \frac{3t^2}{2} + 2t + c$

$v(1) = \frac{3}{2} + 2 + c = 4$

$\frac{3}{2} + c = 4 \quad \boxed{c = \frac{1}{2}}$

$v(t) = \frac{3}{2}t^2 + 2t + \frac{1}{2}$

$x(t) = \int \frac{3}{2}t^2 + 2t + \frac{1}{2}$

$= \frac{t^3}{2} + t^2 + \frac{1}{2}t + C$

$x(1) = \frac{1}{2} + 1 + \frac{1}{2} + C = 6$

$C = 4$

$x(t) = \frac{t^3}{2} + t^2 + \frac{1}{2}t + 4$ D

$x(2) = 4 + 4 + 1 + 4 = \boxed{13}$

25) $y = \sqrt{3+e^x}$ $(0, 2)$

$y' = \frac{1}{2}(3+e^x)^{-1/2}(e^x)$

$y'(0) = \frac{1}{2}(4)^{-1/2}(e^0)$

$y'(0) = \frac{1}{4}$ $m = .25$

$y - 2 = .25(x)$

$y = .25(x) + 2$ B

$y = .25(.08) + 2 = \boxed{2.02}$

$.02$

$$\begin{array}{r} .25 \\ \times\ .08 \\ \hline \end{array}$$

26) B speed is greatest when $|v|$ is greatest (slope)

27) $r = 6$ $\frac{dV}{dt} = 30\pi$ $\frac{dh}{dt} = ?$

$V = \frac{\pi h^2}{3}(18-h)$ $h = 2$

$V = 6\pi h^2 - \frac{\pi h^3}{3}$

$\frac{dV}{dt} = 12\pi h \frac{dh}{dt} - \pi h^2 \frac{dh}{dt}$

$30\pi = 12\pi(2)\frac{dh}{dt} - \pi 4 \frac{dh}{dt}$

$30 = 20 \frac{dh}{dt}$ $\boxed{\frac{dh}{dt} = 1.5}$ C

28) $f(x) = 2x^{5/3} - 5x^{2/3}$

$f'(x) = \frac{10}{3}x^{2/3} - \frac{10}{3}x^{-1/3}$

$\frac{10}{3}x^{-1/3}(x-1) = 0$

$x = 0, 1$

$+$ | $-$ | $+$

inc 0 | 1 inc

E

Venture 2006

Exam I - section I Part B calculator

1) I false
II True slope at 2 = 2
III True (D)

2) $f(x) = \sin^2 x$ $[-\pi/2, \pi/2]$
$g(x) = .5x^2$ Q I, IV
(C)

3) $\int_a^b 2x^2 - x^3$ $f(x)$ $g(x) = x^2 - 2x$

$\int_a^b f(x) > \int_a^b g(x)$

math 9 fnint $(2x^2 - x^3 - (x^2 - 2x),$
$x, a, b) > 0$
II only (B)

4) $y^2 - 3x = 7$
$2y \frac{dy}{dx} - 3 = 0$
$\frac{dy}{dx} = 3/2y = 3(2y)^{-1}$
$\frac{d^2y}{dx^2} = -3(2y)^{-2}(2\frac{dy}{dx})$
$= \frac{-3}{4y^2}(2)(\frac{3}{2y})$
$= \boxed{\frac{-9}{4y^3}}$ (E)

5) I $h(0) = g[f(0)] = g(5) = 0$ False
II $h'(x) = g'[f(x)]f'(x)$
$h'(2) = g'(f(2))f'(2)$
$= g'(1)f'(2)$
$(-)(-) = +$
$h'(2) > 0$, $h(2)$ inc true
III $h'(4) = g'(f(4))f'(4)$ (D)
$g'(2)$ $(+) = 0$ true

*6) $y = e^x$ $y' = e^x$
D

7) $y = 4 - x$ $y = x - \cos x$
$\int_0^{1.858} (4-x) - (x - \cos x)\,dx = 4.93$
(E) ≈ 5

8) $y = 2x + \cos(x^2)$ $0 < x \leq 5$
\# inflection points 8 (C)

9) $\frac{dv}{dt} = \sqrt{1+2t}$
amount melted when $t = 5$
$\int_0^5 \sqrt{1+2^t} = 14.53$ ft (C)

C
*10) $\frac{6-1}{2(5)} = \frac{1}{2}(2 + 2(3) + 2(4) + 2(3) + 2(2) + 1)$
$\frac{1}{2}(27) = 13.5$ (area)
$\pi \int_1^6 13.5$ $13.5x|_1^6$

11) $x(t) = (t+1)(t-3)^3$
vel inc when accel +
$x(t)$ cc up (E)

12) $f(x) = \frac{\ln e^{2x}}{x-1} = \frac{2x}{x-1}$
$y = \frac{2x}{x-1}$ $x = \frac{2y}{y-1}$ $xy - x = 2y$
$\cancel{xy - 2y = x}$
$y(x-2) = x$
$g' = y = \frac{x}{x-2}$
$y'(3) = -2$ (E)

13) $\int \frac{e^{x^2}-2x}{e^{x^2}}$

$= \int 1 - \frac{2x}{e^{x^2}} dx$ $\quad u = -x^2$

$x + \int -2x \cdot e^{-x^2}$ $\quad du = -2x$

$x + e^{-x^2} + C$ C

14) $f(x) = (x+2)^5(x^2-1)^4$

graph 3 B

15) $f(1) = \begin{cases} 1+3b+2 = 3b+3 \\ m+b \end{cases}$

$3b+3 = m+b \rightarrow 2b+3 = m$

$f'(x) = \begin{cases} 3b+4x \\ m \end{cases}$ $\quad f'(1) = \begin{cases} 3b+4 \\ m \end{cases}$

$3b+4 = m$

$2b+3 = m$

$3b+4 = m$

$2b+3 = 3b+4$

$-1 = b$ $\quad m = 1$ B

16)

17) $F(x) + 5 = \int^{x} \sin\left(\frac{\pi t}{4}\right) dt$

$F(x) = \int_2^{x^2} \sin\left(\frac{\pi t}{4}\right) dt - 5$

? $F(2) = \int_2^2 = 0 - 5 = -5$

$F'(2) = \sin\left(\frac{2\pi}{4}\right) - 5 = -4$