AB Calc Solutions

Midterm Exam Review #1 (long paper)

① $f(x) = x^2 - 2x + 3$

$$\lim_{\Delta x \to 0} = \frac{(x+\Delta x)^2 - 2(x+\Delta x) + 3 - (x^2 - 2x + 3)}{\Delta x}$$

$$= x^2 + 2x\Delta x + (\Delta x)^2 - 2x - 2\Delta x + 3 - x^2 + 2x - 3 \,/\, \Delta x$$

$$= 2x\Delta x + (\Delta x)^2 - 2\Delta x \,/\, \Delta x$$

$$= 2x + (\Delta x) - 2$$

$$\lim_{\Delta x \to 0} = \boxed{2x - 2}$$

② $f(x) = \frac{x+1}{x-1}$

$$\frac{\frac{x+\Delta x+1}{x+\Delta x-1} - \frac{x+1}{x-1}}{\Delta x} = \frac{(x+\Delta x+1)(x-1) - (x+1)(x+\Delta x-1)}{(x+\Delta x-1)(x-1)} \cdot \frac{1}{\Delta x}$$

$$= \frac{-2\Delta x}{x^2 + x\Delta x - 2x - \Delta x + 1} \cdot \frac{1}{\Delta x} = \frac{-2}{x^2 + x\Delta x - 2x - \Delta x + 1}$$

$$\lim_{\Delta x \to 0} = \frac{-2}{x^2 - 2x + 1} = \boxed{\frac{-2}{(x-1)^2}}$$

③ $f(x) = x^3 - 3x^2$

$f'(x) = \boxed{3x^2 - 6x}$

④ $f(x) = \frac{2x^3 - 1}{x^2}$

$$f'(x) = \frac{x^2(6x^2) - (2x^3 - 1)(2x)}{x^4}$$

$$= 6x^4 - 4x^4 + 2x \,/\, x^4$$

$$= 2x^4 + 2x \,/\, x^4$$

$$= \boxed{2 + \frac{2}{x^3}}$$

⑤ $g(t) = \frac{2}{3}t^{-2}$

$g'(t) = \boxed{\frac{-4}{3}t^{-3} = \frac{-4}{3t^3}}$

⑥ $f(x) = (1 - x^3)^{1/2}$

$f'(x) = \frac{1}{2}(1 - x^3)^{-1/2}(-3x^2)$

$= \boxed{-3x^2 \,/\, 2\sqrt{1 - x^3}}$

⑦ $f(x) = x^{1/2} - x^{-1/2}$

$\boxed{f'(x) = \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}}$

⑧ $f(x) = \frac{x+1}{x-1}$

$$f'(x) = \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2}$$

$$= x - 1 - x - 1 \,/\, (x-1)^2$$

$$= \boxed{-2 \,/\, (x-1)^2}$$

⑨ $h(x) = 2(3x)^{-2}$

$h'(x) = -4(3x)^{-3}(3) =$

$\boxed{-12 \,/\, (3x)^3}$

⑩ $y = 3\cos(3x+1)$

$y' = -3\sin(3x+1)(3)$

$= \boxed{-9\sin(3x+1)}$

⑪ $y = \frac{1}{2}\csc 2x$

$y' = \frac{-1}{2}\csc 2x \cot 2x\,(2)$

$= \boxed{-\csc 2x \cot 2x}$

⑫ $y = \frac{x}{2} - \frac{\sin 2x}{4}$

$y' = \frac{1}{2} - \frac{\cos 2x}{4}(2)$

$= \frac{1}{2} - \frac{\cos 2x}{2}$

$= \boxed{\frac{1 - \cos 2x}{2}}$

⑬ $y = 1 - \cos 2x + 2\cos^2 x$ $(\cos x)^2$

$y' = \sin 2x\,(2) - 4\cos x \sin x$

$= \boxed{2\sin 2x - 4\cos x \sin x}$

(14) $y = \csc 3x + \cot 3x$

$y' = -\csc 3x \cot 3x (3) - \csc^2 3x (3)$

$= -3\csc 3x \cot 3x - 3\csc^2 3x$

$= \boxed{-3\csc 3x (\cot 3x + \csc 3x)}$

(15) $y = \dfrac{1+\sin x}{1-\sin x}$

$y' = \dfrac{(1-\sin x)(\cos x) - (1+\sin x)(-\cos x)}{(1-\sin x)^2}$

$y' = \cos x - \sin x \cos x + \cos x + \cos x \sin x$

$\boxed{y' = \dfrac{2\cos x}{(1-\sin x)^2}}$

(16) $x^2 + 3xy + y^3 = 10$

$2x + 3x\dfrac{dy}{dx} + 3y + 3y^2\dfrac{dy}{dx} = 0$

$(3x + 3y^2)\dfrac{dy}{dx} = -2x - 3y$

$\boxed{\dfrac{dy}{dx} = -2x - 3y \,/\, 3x + 3y^2}$

(17) $y\sqrt{x} - x\sqrt{y} = 16$

$y\left(\frac{1}{2}x^{-1/2}\right) + \sqrt{x}\dfrac{dy}{dx} - x\left(\frac{1}{2}y^{-1/2}\right)\dfrac{dy}{dx} + \sqrt{y}(-1) = 0$

$\left(\sqrt{x} - \dfrac{x}{2\sqrt{y}}\right)\dfrac{dy}{dx} = \dfrac{-y}{2\sqrt{x}} + \sqrt{y}$

$\dfrac{dy}{dx} = \dfrac{\frac{-y}{2\sqrt{x}} + \sqrt{y}}{\sqrt{x} - \frac{x}{2\sqrt{y}}} = \dfrac{\frac{-y+2\sqrt{xy}}{2\sqrt{x}}}{\frac{2\sqrt{xy}-x}{2\sqrt{y}}} = \dfrac{-y+2\sqrt{xy}}{2\sqrt{x}} \cdot \dfrac{2\sqrt{y}}{2\sqrt{xy}-x} = \boxed{\dfrac{\sqrt{y}(-y+2\sqrt{xy}}{\sqrt{x}(2\sqrt{xy}-x}}$

(18) $x^2 + 9y^2 - 4x + 3y = 0$

$2x + 18y\dfrac{dy}{dx} - 4 + 3\dfrac{dy}{dx} = 0$

$(18y + 3)\dfrac{dy}{dx} = -2x + 4$

$dy/dx = \dfrac{-2x+4}{18y+3} = \boxed{\dfrac{-2(x-2)}{3(6y+1}}$

(19) $y^2 = (x-y)(x^2+y)$

$2y\dfrac{dy}{dx} = (x-y)\left(2x + 1\dfrac{dy}{dx}\right) + (x^2+y)\left(1 - 1\dfrac{dy}{dx}\right)$

$2y\dfrac{dy}{dx} = 2x^2 + x\dfrac{dy}{dx} - 2xy - y\dfrac{dy}{dx} + x^2 - x^2\dfrac{dy}{dx} + y - y\dfrac{dy}{dx}$

$(2y - x + y + x^2 + y)\dfrac{dy}{dx} = 2x^2 - 2xy + x^2 + y$

$\boxed{\dfrac{dy}{dx} = \dfrac{2x^2 - 2xy + x^2 + y}{4y - x + x^2}}$

(20) $y = (x+3)^3 \quad (-2, 1)$

$y' = 3(x+3)^2$

$y' = 3(-2+3) = 3$

tang line $y - 1 = 3(x+2)$

normal line $y - 1 = -\frac{1}{3}(x+2)$

(21) $y = (x-2)^2 \quad (2, 0)$

$y' = 2(x-2)$

$y' = 2(2-2) = 0$

tang line $y - 0 = 0(x-2)$

$y = 0$

normal line $x = 2$