If you find any errors, email me!

AP Calculus Review 4.1 – 4.4

1) $\int_{-1}^{3} (-x^3 + 3x^2 + 1)\,dx = \frac{-x^4}{4} + x^3 + x \Big|_{-1}^{3} =$

$\left(\frac{-81}{4} + 27 + 3\right) - \left(\frac{-1}{4} - 1 - 1\right) = \boxed{12}$

2) $\int_{-2}^{1} (x^4 + x^3 - 4x^2 + 6)\,dx = \frac{x^5}{5} + \frac{x^4}{4} - \frac{4x^3}{3} + 6x \Big|_{-2}^{1} =$

$\left(\frac{1}{5} + \frac{1}{4} - \frac{4}{3} + 6\right) - \left(\frac{-32}{5} + 4 + \frac{32}{3} - 12\right)$

$307/60 - \left(-\frac{56}{15}\right) = 531/60 = \boxed{\frac{177}{20}}$

3) $\int_{-4}^{-1} -\frac{4}{x^3}\,dx = \int_{-4}^{-1} -4x^{-3}\,dx = \frac{-4x^{-2}}{-2} \Big|_{-4}^{-1} = \frac{2}{x^2}\Big|_{-4}^{1} =$

$2 - \left(\frac{1}{8}\right) = \boxed{15/8}$

4) $\int_{-\pi/4}^{-\pi/6} 2\cos x\,dx = 2\sin x \Big|_{-\pi/4}^{-\pi/6} = 2\left(\sin\left(-\frac{\pi}{6}\right) - \sin\left(-\pi/4\right)\right)$

$= 2\left(-\frac{1}{2} - \frac{\sqrt{2}}{2}\right) = \boxed{-1-\sqrt{2}}$

5) $\int_{-4}^{-2} (-x + |-3x-9|)\,dx = \int_{-4}^{-2} -x\,dx + \int_{-4}^{-2} |-3x-9|\,dx$

$-3x - 9 = 0$
$-3x = 9$
$x = -3$

$-\frac{x^2}{2}\Big|_{-4}^{-2} dx + \int_{-4}^{-3} (-3x-9)\,dx + \int_{-3}^{-2} (+3x+9)\,dx$

$-2-(-8) + -\frac{3x^2}{2} - 9x \Big|_{-4}^{-3} \quad \frac{3x^2}{2} + 9x \Big|_{-3}^{-2}$

$6 + \left[\left(\frac{-27}{2} + 27\right) - (-24 + 36)\right] + \left[(6 - 18) - \left(\frac{27}{2} - 27\right)\right]$

$6 + \left[\frac{27}{2} - 12\right] + \left[-12 - \left(-\frac{27}{2}\right)\right]$

$6 + 13.5 - 12 - 12 + 13.5 = \boxed{9}$

6) $\int_{-1}^{3} |4x-1|\,dx = \int_{-1}^{1/4} (-4x+1)\,dx + \int_{1/4}^{3} (4x-1)\,dx$

$= -2x^2 + x \Big|_{-1}^{1/4} + 2x^2 - x \Big|_{1/4}^{3} =$

$\left[\left(-\frac{1}{8} + \frac{1}{4}\right) - (-2-1)\right] + \left[(18-3) - \left(\frac{1}{8} - \frac{1}{4}\right)\right]$

$\left(\frac{1}{8} + 3\right) + \left(15 + \frac{1}{8}\right) = \frac{25}{8} + \frac{121}{8} = \frac{146}{8} = \boxed{\frac{73}{4}}$

Left endpoints - 4 rectangles

1) $y = \frac{x^2}{2} + x + 2 \quad [-5, 3]$ $\qquad \frac{b-a}{n} = \frac{3-(-5)}{4} = 2$ (base)

$-5, -3, -1, 1, \cancel{3}$

$2\left[\left(\frac{25}{2}+5+2\right) + \left(\frac{9}{2}-3+2\right) + \left(\frac{1}{2}-1+2\right) + \left(\frac{1}{2}+1+2\right)\right]$

$= 2\left[\frac{39}{2} + \frac{7}{2} + \frac{3}{2} + \frac{7}{2}\right] = 2(28) = \boxed{56}$

2) $y = x^2 + 3 \quad [-3, 1]$ $\qquad \frac{b-a}{n} = \frac{1-(-3)}{4} = 1$ (base)

$-3, -2, -1, 0, \cancel{1}$

$1(12 + 7 + 4 + 3) = \boxed{26}$

Right endpoints - 5 rectangles

3) $y = -\frac{x^2}{2} + 6 \quad [-3, 2]$ $\qquad \frac{b-a}{n} = \frac{2-(-3)}{5} = 1$

$\cancel{-3}, -2, -1, 0, 1, 2$

$1\left[(-2+6) + \left(-\frac{1}{2}+6\right) + (0+6) + \left(-\frac{1}{2}+6\right) + (-2+6)\right]$

$= 4 + \frac{11}{2} + 6 + \frac{11}{2} + 4 = \boxed{25}$

4) $y = \frac{-x^2}{2} + x + 5 \quad [-1, 4]$ $\qquad \frac{b-a}{n} = \frac{4-(-1)}{5} = 1$

$\cancel{-1}, 0, 1, 2, 3, 4$

$1\left[(0+0+5) + \left(\frac{-1}{2}+1+5\right) + (-2+2+5) + \left(\frac{-9}{2}+3+5\right) + (-8+4+5)\right]$

$= 5 + \frac{11}{2} + 5 + \frac{13}{2} + 1$

$= \boxed{23}$

Average Value

1) $f(x) = -x^2 - 2x + 5 \quad [-4, 0]$ $\qquad \frac{1}{b-a}\int_a^b f(x)\,dx$

$\frac{1}{0-(-4)}\int_{-4}^{0} (-x^2 - 2x + 5)\,dx$

$= \frac{1}{4}\left(-\frac{x^3}{3} - x^2 + 5x\right)\Big|_{-4}^{0}$

$= \frac{1}{4}\left(0 - \left(\frac{64}{3} - 16 - 20\right)\right) = \frac{1}{4}\left(0 - \left(-\frac{44}{3}\right)\right)$

$= \frac{1}{4}\left(\frac{44}{3}\right) = \boxed{\frac{11}{3}}$

1) $f''(x) = 4x \quad f'(2) = 5 \quad f(2) = 12$

$f'(x) = \int 4x\,dx = 2x^2 + C$

$f'(2) = 8 + C = 5 \qquad C = -3$

$f'(x) = 2x^2 - 3$

$f(x) = \int (2x^2 - 3)\,dx = \frac{2x^3}{3} - 3x + C$

$f(2) = \frac{16}{3} - 6 + C = 12$

$C = 38/3$

$\boxed{f(x) = \frac{2x^3}{3} - 3x + 38/3}$

Find $F'(x)$

1) $F(x) = \int_{-2}^{x} \sqrt{3t^2+1}\,dt$

$F'(x) = \sqrt{3x^2+1}$

2) $F(x) = \int_{3}^{-2x^4} (5t-2)\,dt$

$F'(x) = [5(-2x^4) - 2](-8x^3)$

$= (-10x^4 - 2)(-8x^3) = \boxed{80x^7 + 16x^3}$

3) $F(x) = \int_{4x^3}^{2} (2t+1)\,dt = -\int_{2}^{4x^3} (2t+1)\,dt$

$F'(x) = -[2(4x^3) + 1](12x^2)$

$= (-8x^3 - 1)(12x^2) = \boxed{-9x^5 - 12x^2}$

Average Value

2) $f(x) = -x^4 + 2x^2 + 4 \quad [-2, 1]$

$$\frac{1}{1-(-2)} \int_{-2}^{1} (-x^4 + 2x^2 + 4)\,dx = \frac{1}{3}\left(\frac{-x^5}{5} + \frac{2x^3}{3} + 4x\right)\Big|_{-2}^{1}$$

$$= \frac{1}{3}\left[\left(-\frac{1}{5} + \frac{2}{3} + 4\right) - \left(\frac{32}{5} - \frac{16}{3} - 8\right)\right]$$

$$= \frac{1}{3}\left(\frac{67}{15} - \frac{-104}{15}\right) = \frac{1}{3}\left(\frac{171}{15}\right) = \boxed{19/5}$$

Mean Value Theorem

3) $f(x) = -\frac{x^2}{2} + x + \frac{3}{2} \quad [-3, 1]$ $\qquad \int_a^b f(x) = f(c)(b-a)$

$$\int_{-3}^{1} \left(-\frac{x^2}{2} + x + \frac{3}{2}\right) dx = \left(-\frac{c^2}{2} + c + 3/2\right)(1-(-3))$$

$$\left(-\frac{x^3}{6} + \frac{x^2}{2} + \frac{3}{2}x\right)\Big|_{-3}^{1} = \left(-\frac{c^2}{2} + c + \frac{3}{2}\right)(4)$$

$$\left(-\frac{1}{6} + \frac{1}{2} + \frac{3}{2}\right) - \left(\frac{27}{6} + \frac{9}{2} - \frac{9}{2}\right)$$

$$\frac{11}{6} - \frac{27}{6} = \frac{-16}{6}$$

This one was ugly! I won't give you this!

$$-\frac{8}{3}\left(\frac{1}{4}\right) = -\frac{c^2}{2} + c + \frac{3}{2}$$

$$-\frac{2}{3} = -\frac{c^2}{2} + c + \frac{3}{2}$$

$$0 = -\frac{c^2}{2} + c + \frac{13}{6}$$

$$0 = -3c^2 + 6c + 13 \quad \text{doesn't factor}$$

$$c = \frac{-6 \pm \sqrt{36 - 4(-3)(13)}}{-6} = \frac{-6 \pm \sqrt{192}}{-6}$$

$$c = \frac{-6 \pm 8\sqrt{3}}{-6} = \frac{-3 \pm 4\sqrt{3}}{-3} = \boxed{\frac{3 - 4\sqrt{3}}{3}}$$

4) $f(x) = \frac{4}{x^2} \quad [-4, -2]$

$$\int_{-4}^{-2} 4x^{-2}\,dx = \frac{4}{c^2}(-2-(-4))$$

$$-4x^{-1}\Big|_{-4}^{-2} = \frac{4}{c^2}(2)$$

$$-\frac{4}{x}\Big|_{-4}^{-2} = 1 \qquad 1 = \frac{8}{c^2} \qquad c^2 = 8$$

$$\boxed{c = 2\sqrt{2}}$$