Check 14-33

16. $f(x) = x^3 - 2x^2 + 5x - 16$; $f'(x) = 4$

$f'(x) = 3x^2 - 4x + 5$

$3x^2 - 4x + 5 = 4$

$3x^2 - 4x + 1 = 0$

$(3x - 1)(x - 1) = 0$

$\boxed{x = 1/3 \text{ , } 1}$

17. $f(x) = x^3 - 3x$

$3x^2 - 3 = 0$

$x^2 = 1 \quad x = \pm 1$

$\boxed{(1, -2) \ (-1, 2)}$

18. $S(t) = 6t^2 + 240t$

$V(t) = 12t + 240 \quad t = 2$

$12(2) + 240 = \boxed{264 \text{ ft/sec}}$

14. $f(x) = x^2 + 2$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\lim_{h \to 0} \frac{(x+h)^2 + 2 - x^2 - 2}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 2 - x^2 - 2}{h}$

$= \lim_{h \to 0} \frac{h(2x + h)}{h} = \boxed{2x}$

15. $f(x) = 3x^3 + 2x \quad x = 1$

$9x^2 + 2 \quad y = 5$

$f'(x) = 11$

$y - 5 = 11(x - 1)$

$\boxed{y = 11x - 6}$

19. $y = \frac{2x}{(1 - 3x^2)^2}$

$y' = \frac{(1-3x^2)^2(2) - (2x)2(1-3x^2)(-6x)}{(1-3x^2)^4}$

$= \frac{2(1-3x^2)\left[(1-3x^2) + 12x^2\right]}{(1-3x^2)^4}$

$= \boxed{\frac{2(1+9x^2)}{(1-3x^2)^3}}$

20. $y = \frac{1-x}{2-x}$

$y' = \frac{(2-x)(-1) - (1-x)(-1)}{(2-x)^2}$

$= \frac{-2+x+1-x}{(2-x)^2} = \frac{-1}{(2-x)^2}$

$y'' = \frac{((2-x)^2)(0) - (-1)(2(2-x)(-1))}{(2-x)^4}$

$= \frac{-4+2x}{(2-x)^4} = \frac{-2(2-x)}{(2-x)^4}$

$= \boxed{\frac{-2}{(2-x)^3}}$

21. $y = (x^2+x)^{1/3}$

$y' = \frac{1}{3}(x^2+x)^{-2/3}(2x+1)$

$= \boxed{\frac{2x+1}{3(x^2+x)^{2/3}}}$

22. $y = (x^2+2x+5)^6$

$y' = 6(x^2+2x+5)^5(2x+2)$

$12x + 12(x^2+2x+5)^5$

$\boxed{12(x+1)(x^2+2x+5)^5}$

23. $y = (2x+1)^{1/2}(x^3)$

$y' = \left[\frac{1}{2}(2x+1)^{-1/2}(2)\right](x^3) + (3x^2)(\sqrt{2x+1})$

$= \frac{x^3}{\sqrt{2x+1}} + 3x^2\sqrt{2x+1}$

$\frac{x^3 + (3x^2)(2x+1)}{\sqrt{2x+1}}$

$= \frac{x^3 + 6x^3 + 3x^2}{\sqrt{2x+1}} = \frac{7x^3+3x^2}{\sqrt{2x+1}} = \boxed{\frac{x^2(7x+8)}{\sqrt{2x+1}}}$

24. $s = (t^3+1)^2$

$v = 2(t^3+1)(3t^2)$

$= 6t^2(t^3+1)$

$a = (6t^2)(3t^2) + (t^3+1)(12t)$

$t = 1$

$(6(1)^2)(3(1)^2) + (1^3+1)(12(1))$

$= (6)(3) + (2)(12)$

$= 18 + 24 = \boxed{42 \text{ ft/sec}^2}$

25. $y = \frac{x}{x+y}$

$y' = \frac{(x+y)(1) - (x)(1+y')}{(x+y)^2}$

$y' = \frac{x+y-(x)(1+y')}{(x+y)^2}$

$(x+y)^2 y' = x+y-x-xy'$

$(x+y)^2 y' + xy' = y$

$y'((x+y)^2 + x) = y$

$\boxed{y' = \frac{y}{(x+y)^2 + x}}$

26. $x^2 + xy + y^2 = 5$

$2x + (x)(y') + (y)(1) + 2y \cdot y' = 0$

$= xy' + 2y \cdot y' = -2x - y$

$y'(x + 2y) = -2x - y$

$$y' = \frac{-2x - y}{x + 2y}$$

27. $y^4 - xy^2 = x \quad (1/2, 1)$

$4y^3 \cdot y' - (x)(2y\,y') - (y^2)(1) = 1$

$4y^3 \cdot y' - x(2y \cdot y') = y^2 + 1$

$y'(4y^3 - 2yx) = y^2 + 1$

$$y' = \frac{y^2 + 1}{4y^3 - 2xy}$$

$$y' = \frac{(1)^2 + 1}{4(1)^3 - 2(1)(1)}$$

$$= \frac{2}{4 - 1} = \boxed{\frac{2}{3}}$$

28. $\frac{dr}{dt} = 5 \quad \frac{dA}{dt} = ? \quad r = 10$

$A = \pi r^2$

$\frac{dA}{dt} = \pi \left[2r \frac{dr}{dt} \right]$

$= \pi [2(10)(5)]$

$= \boxed{100\pi \text{ in}^2/\text{min}}$

29. $\frac{dV}{dt} = 28 \quad \frac{dr}{dt} = ? \quad r = 3$

$\frac{dV}{dt} = \frac{4\pi}{3} \left[3r^2 \frac{dr}{dt} \right]$

$28 = \frac{4\pi}{3} \left[3(3)^2 \frac{dr}{dt} \right]$

$28 = \frac{dr}{36\pi} \frac{dr}{dt} = \boxed{\frac{7}{9\pi} \text{ ft/min}}$

30. $10y = -x^2 + 4x - 3$

$10\,dy/dt = -2x + 4\,dx/dt$

$10(-3) = -2(5) + 4\,dx/dt$

$-30 = -6\,dx/dt$

$\boxed{dx/dt = 5}$

31. $f(x) = x(2x+1)^{1/2}$

$(x)\left[\frac{1}{2}(2x+1)^{-1/2}(2)\right] + \sqrt{2x+1}$

$= \frac{x}{\sqrt{2x+1}} + \sqrt{2x+1}$

$= \frac{x + 2x + 1}{\sqrt{2x+1}} = \frac{3x+1}{\sqrt{2x+1}} = 0$

$\boxed{x = -1/3 \quad x = -1/2}$

32. $f(x) = 3x^4 - 4x^3$

$12x^3 - 12x^2 = 0$

$12x^2(x - 1) = 0$

$\boxed{x = 0, 1}$

33. $f(x) = x^2 - 2x + 1 \quad f'(x) = 2x - 2 = 0 \quad x = 1$

minimum: (1, 0)
maximum: (3, 4)