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Pd 2

1st Semester Final Review

1. $\lim_{x\to 0^-} f(x) = 0$

$\lim_{x\to 0^+} f(x) = 1$

$\boxed{\therefore \lim_{x\to 0} f(x) \text{ DNE}}$

2. $\lim_{x\to 1} \frac{x^2+x-2}{x-3}$

$= \lim_{x\to 1} \frac{(x-1)(x+2)}{x-3}$

$= \boxed{0}$

3. $\lim_{x\to 1} f(x)$ $f(x) = \begin{cases} x^2+4, & x\neq 1 \\ 2, & x=1 \end{cases}$

$\boxed{\lim_{x\to 1} f(x) = 5}$

4. $\lim_{x\to 1} \frac{x^2+x-2}{x-1}$

$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x+2)}{\cancel{x-1}} = \boxed{3}$

5. $\lim_{x\to 1} \frac{1-\sqrt{2x^2-1}}{x-1} \cdot \frac{1+\sqrt{2x^2-1}}{1+\sqrt{2x^2-1}}$

$= \frac{1-2x^2+1}{(x-1)(1+\sqrt{2x^2-1})}$

$= \frac{2-2x^2}{}$

$\frac{2(1+x)(\cancel{1-x})}{(\cancel{x-1})(1+\sqrt{2x^2-1})}$

$\frac{-2(1+x)}{1+\sqrt{2x^2-1}}$

$= -4/2 = \boxed{-2}$

6. $f(x) \begin{cases} x^2+1 & x\leq 0 \\ 2x-3 & x>0 \end{cases}$

a. $\lim_{x\to 0^-} f(x) = \boxed{1}$

b. $\lim_{x\to 0^+} f(x) = \boxed{-3}$

c. $\lim_{x\to 0} f(x) = \boxed{\text{DNE}}$

7. $f(x) = \frac{x-2}{x^2-4}$

$= \frac{x-2}{(x-2)(x+2)}$

$\boxed{\begin{array}{l}\text{removeable: } x=2 \\ \text{nonremoveable: } x=-2\end{array}}$

8. $f(x) = \frac{5}{x-1}$ $g(x) = x^4$

a. $f(g(x))$

$= \boxed{\frac{5}{x^4-1}}$

b. $\boxed{\text{discontinuous at } x = \pm 1}$

9. $f(x) = \begin{cases} x^2 & x\leq 3 \\ c/x, & x>3 \end{cases}$

$\frac{c}{3} = 9$ $\boxed{c=27}$

10. $\lim_{x\to 3} \frac{3}{x^2-6x+9}$ $\boxed{\infty}$

11. $f(x) = \dfrac{x^2+3x-1}{x+7}$

$\boxed{VA: x=-7}$

12. $f(x) = \dfrac{x^2-x-2}{x^2+x-6}$

$= \dfrac{(x-2)(x+1)}{(x+3)(x-2)}$

$\boxed{VA: x=-3}$

13. a. undefined
b. negative
c. zero
d. positive
e. zero

14. $f(x) = x^2+2$

$\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$\lim\limits_{h\to 0} \dfrac{(x+h)^2+2-x^2-2}{h}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{x^2}+2xh+h^2\cancel{+2}\cancel{-x^2}\cancel{-2}}{h}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{h}(2x+h)}{\cancel{h}} = \boxed{2x}$

15. $f(x) = 3x^3+2x, \quad x=1$

$9x^2+2 \qquad y=5$

$f'(x) = 11$

$y-5 = 11(x-1)$

$\boxed{y = 11x-6}$

16. $f(x) = x^3-2x^2+5x-16, \quad f'(x)=4$

$f'(x) = 3x^2-4x+5$

$3x^2-4x+5=4$

$3x^2-4x+1=0$

$(3x-1)(x-1)=0$

$\boxed{x = 1/3, 1}$

17. $f(x) = x^3-3x$

$3x^2-3=0$

$x^2=1 \quad x=\pm 1$

$\boxed{(1,-2)\ (-1,2)}$

18. $S(t) = 6t^2+240t$

$v(t) = 12t+240 \quad t=2$

$12(2)+240 = \boxed{264 \text{ ft/sec}}$

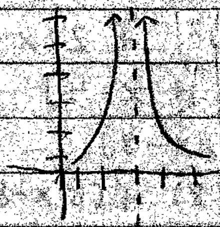
19. $y = \dfrac{2x}{(1-3x^2)^2}$

$y' = \dfrac{(1-3x^2)^2(2) - (2x)2(1-3x^2)(-6x)}{(1-3x^2)^4}$

$= \dfrac{2(1-3x^2)(1-3x^2+12x^2)}{(1-3x^2)^4}$

$= \boxed{\dfrac{2(1+9x^2)}{(1-3x^2)^3}}$

34. a. $f(x) = \frac{1}{(x-3)^2}$



b $f(2) = 1$

$f(4) = 1$

c. Rolles theorem does not apply to f on the interval [2, 4] because f is not continuous

35. $f(x) = x^4 - 4x^3 + 4x^2 + 1$

$f(3) = 81 - 108 + 36 + 1 = 10$

$f(-1) = 1 + 4 + 4 + 1 = 10$

$f'(x) = 4x^3 - 12x^2 + 8x = 0$

$4x(x^2 - 3x + 2) = 0$

$x = 0 \quad (x-1)(x-2)$

$\boxed{x = 0, 1, 2}$

36. $f(x) = 3x - x^2$

$f'(x) = 3 - 2x$

$3 - 2x = \frac{f(3) - f(2)}{1}$

$3 - 2x = 0 - 2$

$3 - 2x = -2$

$-2x = -5$

$\boxed{x = 5/2}$

37. $f(x) = \frac{1}{x^2} = x^{-2}$ $f'(x) = -2x^{-1} = -2/x$

$-2/x = 0 \quad x = 0$

$+ \quad -$

0

$\boxed{\text{inc: } (-\infty, 0) \quad \text{dec: } (0, \infty)}$

38. $f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x = 0$

$3x(x-2) = 0 \quad x = 0, 2$

$+ \quad - \quad +$

$0 \quad 2$

$\boxed{\text{inc: } (-\infty, 0) \cup (2, \infty) \quad \text{dec: } (0, 2)}$

39. dec: $(-\infty, 2) \cup (2, \infty)$

40. $y = \frac{2x}{(x+4)^3}$ $y' = \frac{(x+4)^3(2) - (2x)[3(x+4)^2]}{((x+4)^3)^2}$

$\frac{2(x+4) - (2x)(3)}{(x+4)^4} = \frac{2x + 8 - 6x}{(x+4)^4}$

$= \frac{8 - 4x}{(x+4)^4} = 0 \quad x = -4, 2$

$+ \quad + \quad -$

$-4 \quad 2$

$\boxed{\max\left(2, \frac{1}{54}\right)}$

$-16 + 12 + 24$

41. $f(x) = 2x^3 + 3x^2 - 12x$

$6x^2 + 6x - 12 = 0$

$6(x^2 + x - 2) = 0 \quad x = 1, -2$

$+ \quad - \quad +$

$-2 \quad 1$

$\boxed{\min(1, -7) \quad \max(-2, 20)}$

42. $f(x) = -x^4 + 2x^3$

$f'(x) = -4x^3 + 6x^2$

$-2x^2(2x - 3) = 0$

$x = 0, 3/2$

$+ \quad + \quad -$

$0 \quad 3/2$

$\boxed{x = 3/2}$

43. $f(x) = \dfrac{x}{1-x}$

$f'(x) = \dfrac{(1-x) - (x)(-1)}{(1-x)^2}$

$\dfrac{\cancel{1-x+x}\ 1}{(1-x)^2} = 0$

$\boxed{\text{no critical \#'s} \quad 1 \text{ is a V.A.}}$

44. relative maximum

$16 - 32 + 2$

45. $f(x) = x^4 - 4x^3 + 2$

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x = 0$

$12x(x - 2) = 0$

$x = 0, 2$

$f'' \quad + \quad - \quad +$

$f \quad 0 \quad 2$

$\boxed{\text{ccu: } (-\infty, 0) \cup (2, \infty) \quad \text{ccd: } (0, 2) \quad \text{POI: } (0, 2)\ (2, -14)}$

46. $f(x) = 2x(x-4)^3$

$f'(x) = (2x)[3(x-4)^2] + (x-4)^3(2)$

$= 6x(x-4)^2 + 2(x-4)^3$

$2(x-4)^2\ [3x + x - 4]$

$= (x-4)^2\ (8x - 8)$

$f''(x) = (x-4)^2(8) + (8x-8)[2(x-4)]$

$(x-4)\ [(x-4)(8) + (8x-8)(2)]$

$= (x-4)[(8x - 32) + (16x - 16)]$

$= (x-4)[24x - 48] = 0$

$x = 2, 4$

$+ \quad - \quad +$

$2 \quad 4$

$\boxed{\text{POI } (2, -32) \quad (4, 0)}$

47. $f(x) = x^3 - 3x^2 - x + 7$

$f'(x) = 3x^2 - 6x - 1$

$f''(x) = 6x - 6 = 0$

$x = 1$

$- \quad +$

$\boxed{\text{POI: } (1, 4)}$

48. $f(x) = x^3 - x^2 + 3$

$f'(x) = 3x^2 - 2x = 0$

$x(3x - 2) = 0 \quad x = 0, 2/3$

$f''(x) = 6x - 2$

$\boxed{f''(0) = -2 \text{ max} \quad f''(2/3) = 2 \text{ min}}$