AP Calculus AB – 1st semester practice multiplechoice

Key

Limits

1. $\lim_{x \to -2} \dfrac{-3x-6}{x^2+x-2}$ is

-plug in - you get 0 in denom.
-factor $\dfrac{-3(\cancel{x+2})}{(\cancel{x+2})(x-1)} = \dfrac{-3}{x-1}$ plug in $\dfrac{-3}{-3} =$

A. -1 B. $\dfrac{-1}{3}$ C. $\dfrac{1}{3}$ $\boxed{\text{D. } 1}$ E. Does not exist

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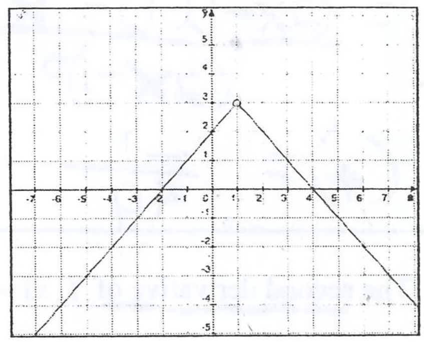
2. The figure to the right shows the graph of $f(x)$.
Which of the following statements are true?



I. $\lim_{x \to 1^-} f(x)$ exists $=3$ True (limit can be a hole)

II. $\lim_{x \to 1^+} f(x)$ exists $=3$ True

III. $\lim_{x \to 1} f(x)$ exists True

A. I only B. II only C. I and II only $\boxed{\text{D. I, II and III}}$ E. none are true

Limit exists but f(x) is not continuous. You would also need f(1) = 3 in order to have continuity

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3. Let $f(x) = \begin{cases} \dfrac{x^2-6x+9}{x-3} & \text{if } x \neq 3 \\ \sqrt{3x} & \text{if } x = 3 \end{cases}$ $\dfrac{(x-3)(\cancel{x-3})}{\cancel{x-3}}$ hole at $x=3$ $(3,0)$

Which of the following statements I,II, and III are true?

✓ I. $\lim_{x \to 3} f(x)$ exists $=0$ ✓ II. $f(3)$ exists $=3$ III. f is continuous at $x = 3$ no Since I ≠ II

$x \to 3^+ = x \to 3^-$

A. only I B. only II $\boxed{\text{C. I and II}}$ D. none of them E. all of them

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4. $\lim_{x \to -\infty} \dfrac{-4-x-x^2}{2x^2+3x-2} =$ is $\dfrac{-1}{2}$

A. 0 B. 2 C. -2 $\boxed{\text{D. } \dfrac{-1}{2}}$ E. ∞

5. If $f(x)=\left(1-4x^2\right)^4$, then $f'(x)=$ chain rule

A. $32x\left(1-4x^2\right)^3$ B. $4(1-8x)^3$ C. $-32(1-8x)^3$ D. $4\left(1-4x^2\right)^3$ $\boxed{\text{E. } -32x\left(1-4x^2\right)^3}$

$f'(x)=4(1-4x^2)^3(-8x)=-32x(1-4x^2)^3$

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6. The slope of the tangent line to $y=\dfrac{x}{2x-1}$ at $x=4$ is quotient rule, plug in

A. $\dfrac{1}{2}$ B. $\dfrac{-1}{14}$ C. $\dfrac{1}{14}$ D. $\dfrac{1}{49}$ $\boxed{\text{E. } \dfrac{-1}{49}}$

$y'=\dfrac{(2x-1)(1)-x(2)}{(2x-1)^2}=\dfrac{-1}{(2x-1)^2}$

$y'(4)=\dfrac{-1}{49}$

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7. The second derivative of $f(x)=\sin x\cos x$ is product rule $f'(x)=\sin x(-\sin x)+\cos x(\cos x)$ $=-\sin^2 x+\cos^2 x$

A. 0 $\boxed{\text{B. } -4\sin x\cos x}$ C. $(\cos x)^2-(\sin x)^2$
D. $2\cos x-2\sin x$ E. $2\sin x-2\cos x$

chain $f''(x)=-2\sin x\cos x-2\cos x\sin x$ $=-4\cos x\sin x$

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8. If $f(-2)=4$ and $f'(-2)=-1$, find the derivative of $\dfrac{f(x)}{x^2}$ at $x=-2$. quotient $\dfrac{x^2 f'(x)-f(x)(2x)}{x^4}$

point $(-2,4)$ slope $=-1$ plug in $\dfrac{4(-1)-4(-4)}{16}=\dfrac{-4+16}{16}$ $\dfrac{12}{16}$

$\boxed{\text{A. } \dfrac{3}{4}}$ B. 1 C. $\dfrac{1}{4}$ D. $-\dfrac{1}{4}$ E. $\dfrac{1}{2}$

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need point & slope $y(\pi/4)=\sec\pi/4=\frac{2}{\sqrt{2}}=\sqrt{2}$

9. Find the equation of the line tangent to $y=\sec x$ at $x=\dfrac{\pi}{4}$. $y'=\sec x\tan x$ $y'(\pi/4)=\sqrt{2}$

A. $y-\dfrac{\sqrt{2}}{2}=\sqrt{2}\left(x-\dfrac{\pi}{4}\right)$ $\boxed{\text{B. } y-\sqrt{2}=\sqrt{2}\left(x-\dfrac{\pi}{4}\right)}$ C. $y=\sqrt{2}\left(x-\dfrac{\pi}{4}\right)$

D. $y-\sqrt{2}=x-\dfrac{\pi}{4}$ E. $y-\sqrt{2}=-\left(x-\dfrac{\pi}{4}\right)$ $y-\sqrt{2}=\sqrt{2}(x-\pi/4)$

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calculator same slope - set slopes =

10. For how many values of x are the tangent lines to $y=\sin x+\cos x$ and $y=\dfrac{x^3}{12}-\dfrac{x^2}{2}-\dfrac{x}{4}$ $\boxed{\text{parallel?}}$

$y'=\cos x-\sin x$ $y'=\dfrac{x^2}{4}-x-\dfrac{1}{4}$

A. 0 B. 1 C. 2 D. 3 E. 4

$\cos x-\sin x=\dfrac{x^2}{4}-x-1/4$ see how many intersections

11. Let f, g and their derivatives be defined by the table below. If the derivatives of $f(g(x))$ and $g(f(x))$ are equal at $x = 3$, what is the value of a?

$f'(g(x))g'(x) = g'(f(x))f'(x)$

$f'(g(3))g'(3) = g'(f(3))f'(3)$

$f'(4)(2) = g'(1)(4)$

$2(2) =$

$4 = a(4)$

$a = 1$

x	1	2	3	4
$f(x)$	3	2	1	4
$g(x)$	2	1	4	3
$f'(x)$	4	3	4	2
$g'(x)$	a	1	2	3

A. 1 B. 2 C. 3 D. 4 E. Impossible

Implicit Differentiation

12. If $x^2 + xy - y = 7$, find $\frac{dy}{dx}$ at $(3,-1)$

$2x + x\frac{dy}{dx} + y(1) - \frac{dy}{dx} = 0$

$2(3) + 3\frac{dy}{dx} + (-1)(1) - \frac{dy}{dx} = 0$

$2\frac{dy}{dx} = -5$

$\frac{dy}{dx} = -5/2$

A. -3 B. 1 C. $-\frac{5}{2}$ D. -1 E. $\frac{1}{2}$

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13. At which points is the tangent line to the curve $8x^2 + 2y^2 = 6xy + 14$ vertical?

Slope undefined

$16x + 4y\frac{dy}{dx} = 6x\frac{dy}{dx} + 6y$

$(4y - 6x)\frac{dy}{dx} = 6y - 16x$

$\frac{dy}{dx} = \frac{6y - 16x}{4y - 6x}$

$4y - 6x = 0$

I. (-2, -3) II (3, 8) III. (4, 6)

A. I only B. II only C. III only D. I and II only E. I and III only

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Continuity and Differentiability

14. If $f(x) = \begin{cases} x^2 - 5x + 8, x \geq 3 \\ x - 1, x < 3 \end{cases}$, describe the behavior of the graph of $f(x)$ at $x = 3$

I. $\lim\limits_{x \to 3} f(x)$ exists II. $f(x)$ is continuous at $x = 3$ III. $f(x)$ is differentiable at $x = 3$

a) I only b) II only c) I and II only d) I, II and III e) none are true

continuous at 3

$\lim\limits_{x \to 3^+} = 2$ $\lim\limits_{x \to 3^-} = 2$ $f(3) = 2$

differentiable at 3

$f'(x) = \begin{cases} 2x - 5 = 1 \\ 1 = 1 \end{cases}$

$-16\frac{81}{16}$

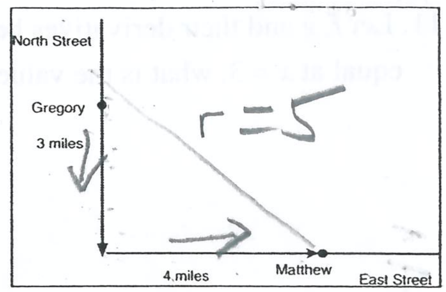
$-81 + 18(9) + 30$

162

111

Related Rates $x^2+y^2=r^2$ $\quad 2x\frac{dx}{dt}+2y\frac{dy}{dt}=2r\frac{dr}{dt}$

15. Matthew is visiting Gregory at his home on North Street. Shortly after Matthew leaves, Gregory realizes that Matthew left his wallet and begins to chase him. When Gregory is 3 miles from the 90° intersection along North Street traveling at 45 mph towards the intersection, Matthew is 4 miles along East street traveling away from the intersection at 30 mph. At that time, how fast is the distance between the two men changing?



formula -> deriv -> plug in $\quad \frac{dr}{dt}=-3$

$y=3 \quad x=4$

$\frac{dy}{dt}=-45 \quad \frac{dx}{dt}=30$

A. getting closer at 3 mph
B. getting further away at 51 mph
C. getting closer at 51 mph
D. Getting closer at 15 mph
E. getting closer at 18 mph

$\frac{dh}{dt}=3$ 16. A cylinder has both its height and radius changing. Its height is increasing at the rate of 3 meters/min. When the height of the cylinder is 8 meters and its radius is 2 meters, the volume is not changing. How fast is the radius decreasing in meters/min? (The volume of a cylinder is given by $V=\pi r^2 h$.)

$h=8$
$r=2$
$\frac{dV}{dt}=0$
$\frac{dr}{dt}=?$ you should get a negative

$\frac{dV}{dt}=\pi r^2\frac{dh}{dt}+h2\pi r\frac{dr}{dt}$
$0=\pi 4(3)+32\pi\frac{dr}{dt}$
$0=12\pi+32\pi\frac{dr}{dt}$
$-12\pi/32\pi$

A. 3 B. $\frac{3}{8}$ C. 6 D. 3π E. 6π

Function Analysis

17. Given that $f(x)=x^3$ find all values of x in the interval $(-1,1)$ that satisfy the mean value theorem.

a b

A. 0 B. $\sqrt{\frac{1}{3}}$ C. $\boxed{\pm\sqrt{\frac{1}{3}}}$ D. 1 E. ± -1

$f'(x)=\frac{f(b)-f(a)}{b-a}$

$3x^2=\frac{1-(-1)}{1-(-1)}$

$3x^2=1$
$x^2=1/3$
$x=\pm\sqrt{1/3}$

check that answers are in interval

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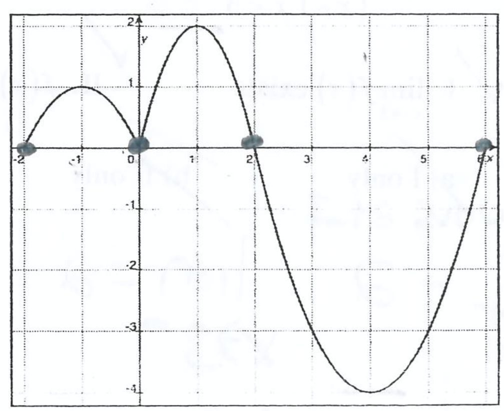
18. The graph of f', the derivative of f is shown to the right for $-2 \le x < 6$. At what values of x does f have a horizontal tangent line?

slope = 0



A. $x=0$ only
B. $x=-1, x=1, x=4$
C. $x=2$ only
D. $x=-2, x=2, x=6$
E. $x=-2, x=0, x=2, x=6$

$f''>0$

19. Given $f(x)=2-\frac{x^3}{6}-x^2$. On what interval(s) is the graph of f concave upwards?

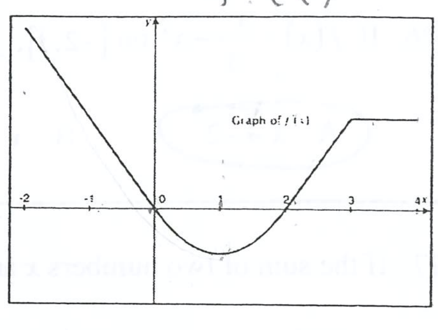
A. $(-\infty,-2)$ B. $(-\infty,2)$ C. $(-2,\infty)$ D. $(2,\infty)$ E. $(-2,2)$

$f'(x)=\frac{-3x^2}{6}-2x$ $\quad f''(x)=-x-2=0$
$x=-2$

$+ \quad - $
-2

20. The graph of $f'(x)$, the derivative of f, is shown to the right. Which of the following statements is **not** true?

$f'(t)$



T A. f is increasing on $2 < x \le 4$. f' pos

F (B) f has a local minimum at $x = 1$. f' changes $-$ to $+$

T C. f has a local maximum at $x = 0$. f' changes $+$ to $-$

T D. f has an inflection point at $x = 1$. f'' changes signs $\rightarrow$ f' changes inc/dec or dec/inc

T E. f is concave down on $-2 \le x < 1$. f'' neg $\rightarrow$ f' dec

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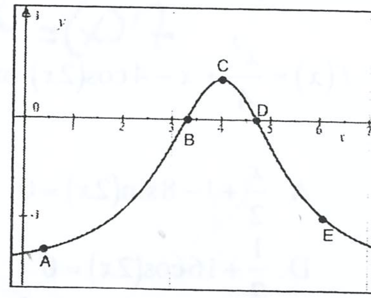
21. The graph of $y = f(x)$ is to the right. At what point are both $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} > 0$?

choices?

$f(x)$ inc $\quad$ $f(x)$ ccu

A



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22. If $f(x) = \frac{1}{x^n}$, where $n > 0$ and $x > 0$, describe the concavity of the graph of $f(x)$.

$\frac{1}{x^1} =$ $\quad$ $\frac{1}{x^{.5}}$ $\quad$ $\frac{1}{x^2}$

(A) Always concave up

B. Always concave down

~~C. Concave down if $n < 1$, concave up if $n \ge 1$~~

~~D. Concave up if $n < 1$, concave down if $n \ge 1$~~

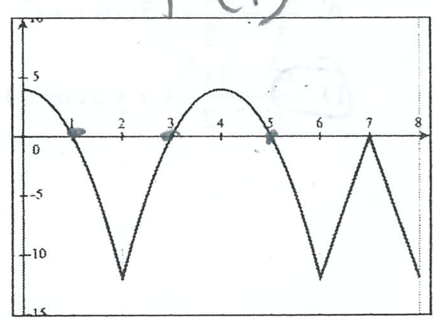
~~E. Concave up if $n < 2$, concave down if $n \ge 2$~~

Straight-Line Motion

23. A locomotive is moving along a straight track. Its velocity v of the locomotive at time t, $0 \le t \le 8$ is given by the function whose graph is to the right. At what value of t does the locomotive change direction?

when f' changes signs

$f'(t)$



A. 4 only $\quad$ B. 2 and 4 only $\quad$ C. 2, 4 and 6 only

(D.) 1, 3, and 5 only $\quad$ E. 1, 3, 5 and 7 only

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24. A particle moves along a horizontal line with position $x(t) = \frac{10}{t}$. Describe its motion at $t = 1$.

$x(t) = 10t^{-1}$ $\quad$ $v(t) = -10t^{-2} = \frac{-10}{t^2}$

A. Moving right and slowing down

B. Moving right and speeding up

C. Moving left and slowing down

(D) Moving left and speeding up

E. Stopped

$t = 0$ at 1 moving left

$a(t) = 20t^{-3} = \frac{20}{t^3}$ at 1 speeding up

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25. A ball is thrown straight up from the top of a hill 30 feet high with initial velocity of 72 ft/sec. How high above level ground will the ball get? (objects subjected to gravity adhere to $s(t) = -16t^2 + v_0 t + s_0$ where s is the height of the object in feet, v_0 is the initial velocity and s_0 is the initial height).

$s(t) = -16t^2 + 72t + 30$

$v(t) = -32t + 72 = 0$

$t = \frac{72}{32} = \frac{9}{4}$ reaches max

A. 72 sec $\quad$ B. 81 feet $\quad$ C. 88 feet $\quad$ (D. 111 ft) $\quad$ E. 144 ft

$s(9/4) = -16(9/4)^2 + 72(9/4) + 30 = 111$

Optimization

$f'(x) = x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$

$f(0) = 0$

$f(2) = -4/3$

$f(-2) = -20/3$

$f(3) = 0$

26. If $f(x) = \dfrac{x^3}{3} - x^2$ on $[-2,3]$, $f(x)$ has an absolute minimum at

A. $x = -2$ B. $x = 0$ C. $x = 2$ D. $x = 3$ E. $x = 0$ and $x = 2$

$9-9$

$x + y = 12$ $y = 12 - x$ $x^3(12-x)$ $12x^3 - x^4$

$36x^2 - 4x^3 = 0$

27. If the sum of two numbers x and y is 12, what is the maximum product of x^3y?

A. 9 B. 27 C. 729 D. 2187 E. 19683

$x = 9$ $y = 3$ $4x^2(9-x) = 0$ $x = 0, 9$

$f'(x) = \frac{x}{2} + 1 + 4\sin(2x)(2) = \frac{x}{2} + 1 + 8\sin(2x) = 0$

28. $f(x) = \dfrac{x^2}{4} + x - 4\cos(2x)$ on $[0, 2\pi]$ has a possible maximum slope at the x-value that solves the equation

A. $\dfrac{x}{2} + 1 - 8\sin(2x) = 0$ B. $\dfrac{x}{2} + 1 + 8\sin(2x) = 0$ C. $\dfrac{x}{2} + 1 + 4\sin(2x) = 0$

D. $\dfrac{1}{2} + 16\cos(2x) = 0$ E. $\dfrac{1}{2} + 4\cos(2x) = 0$

Indefinite Integration

$\frac{x^3}{3} - \frac{3x^2}{2} + x + 5\cos x + C$

29. $\int \left(x^2 - 3x + 1 - 5\sin x\right) dx$

A. $\dfrac{x^3}{3} - \dfrac{3x^2}{2} - 5\cos x + C$ B. $\dfrac{x^3}{3} - \dfrac{3x^2}{2} + 5\cos x + C$ C. $\dfrac{x^3}{3} - \dfrac{3x^2}{2} + x - 5\cos x + C$

D. $\dfrac{x^3}{3} - \dfrac{3x^2}{2} + x + 5\cos x + C$ E. $2x - 3 - 5\cos x + C$

$f'(x) = x^8 - 2x^5 + x^2$

$f(x) = \int f'(x) = \frac{x^9}{9} - \frac{x^6}{3} + \frac{x^3}{3} + C$

30. If $f'(x) = \left(x^4 - x\right)^2$ and $f(1) = 1$, find $f(x)$

A. $\dfrac{x^9}{9} + \dfrac{x^3}{3} + \dfrac{5}{9}$ B. $\dfrac{x^9}{9} - \dfrac{x^6}{6} + \dfrac{x^3}{3} + \dfrac{13}{18}$ C. $\dfrac{x^9}{9} - \dfrac{x^6}{3} + \dfrac{x^3}{3} + \dfrac{8}{9}$

D. $\dfrac{x^9}{9} - \dfrac{x^6}{3} + \dfrac{x^3}{3} + \dfrac{1}{9}$ E. $\dfrac{x^9}{9} - \dfrac{x^6}{3} + \dfrac{x^3}{3} + 1$

$\frac{1}{9} - \frac{1}{3} + \frac{1}{3} + C = 1$

$C = 8/9$

$f(x) = \frac{x^9}{9} - \frac{x^6}{3} + \frac{x^3}{3} + \frac{8}{9}$