A.P. Calculus **Midterm** Review **Name** KEY

Directions: Choose the best answer. Calculators allowed. Leave answers blank if you do not know the answer. Good luck.

1) $\lim\limits_{x\to -5}\dfrac{2x+10}{x^2+2x-15}$ is

$\dfrac{2(x+5)}{(x+5)(x-3)} = \dfrac{2}{x-3}$ $\lim\limits_{x\to -5}\dfrac{2}{x-3} = \dfrac{2}{-8} = \dfrac{-1}{4}$

a) 0 b) $\dfrac{-1}{8}$ c) $\dfrac{-1}{4}$ d) ∞ e) none of these

---

2) $\lim\limits_{x\to 1}\dfrac{x^3-1}{x^2-1}$ is

$\dfrac{\cancel{(x-1)}(x^2+x+1)}{(x+1)\cancel{(x-1)}}$ $\lim\limits_{x\to 1}\dfrac{x^2+x+1}{x+1} = \dfrac{3}{2}$

a) $\dfrac{3}{2}$ b) $\dfrac{1}{2}$ c) 0 d) ∞ e) DNE

---

3) $\lim\limits_{x\to 4}\dfrac{-3x+1}{(x-4)^2}$ is

$\dfrac{-11}{0}$ $\dfrac{-}{+} = -\infty$

$\infty, -\infty$, DNE

$\dfrac{\text{constant}}{0}$

a) -11 b) -13 c) ∞ d) $\boxed{-\infty}$ e) DNE

---

4) Let $f(x)=\begin{cases}\dfrac{4-x^2}{x-2} & \text{if } x \neq 2\\ 4 & \text{if } x = 2\end{cases}$ Which of the following statements I,II, and III are true?

$\dfrac{(2+x)(2-x)}{x-2} = -(2+x)$

$\lim\limits_{x\to 2^+} = -4$ $\lim\limits_{x\to 2^-} = -4$

I. $\lim\limits_{x\to 2} f(x)$ exists II. $f(2)$ exists $= 4$ III. f is continuous at $x = 2$

a) only I b) only II c) I and II d) none of them e) all of them

---

5) If $\begin{cases} f(x) = \dfrac{x^2-6x}{x} & \text{for } x \neq 0\\ f(0) = 2k-1 \end{cases}$

$\begin{cases} x-6 & \text{for } x\neq 0\\ 2k-1 & \text{for } x=0\end{cases}$

continuous

1) $f(0)$ exists
2) $\lim\limits_{x\to 0^+} = \lim\limits_{x\to 0^-}$
3) $f(0) = \lim\limits_{x\to 0}$

and if f is continuous at $x = 0$, then k =

$2k-1=-6$
$2k=-5$
$k=-5/2$

a) -6 b) $\dfrac{-5}{2}$ c) 0 d) $\dfrac{5}{2}$ e) 6

6) If $f(x) = x^2 - 1$ and $g(x) = \frac{1}{x}$, which of the following statements is FALSE?

a) $f(x)$ is continuous everywhere true

b) $g(x)$ is continuous except at $x = 0$ true

c) $f(g(x))$ is continuous except at $x = 0$ true $\left(\frac{1}{x}\right)^2 - 1 = \frac{1}{x^2} - 1 = \frac{1-x^2}{x^2}$

d) $g(f(x))$ is continuous except at $x = 1$ $\frac{1}{x^2-1}$ false not continuous at $x = \pm 1$

e) All are true

VA $x = \pm 1$

---

7) The normal (perpendicular) line to the curve $y = \sqrt{8 - x^2}$ at (-2, 2) has slope

$y' = \frac{1}{2}(8-x^2)^{-1/2}(-2x)$

$y' = \frac{-x}{\sqrt{8-x^2}}$

a) -2 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) 1 e) -1

$y'(-2) = \frac{2}{\sqrt{4}} = 1$

$\perp$ slope $= -1$

---

8) If $f(x) = \frac{256}{\sqrt{x}} + 64\sqrt{x} + 3\sqrt[3]{x^2}$, then $f'(64) =$

a) 4.25 b) 8.75 c) 0.75 d) 10.25 e) 5.78

$f(x) = 256x^{-1/2} + 64x^{1/2} + 3x^{2/3}$

$f'(x) = -128x^{-3/2} + 32x^{-1/2} + 2x^{-1/3}$

$f'(64) = -\frac{1}{4} + 4 + \frac{1}{2} =$

---

9) If the graph of the second derivative of some function f is a line of slope -6, then f could be which type of function?

$y'' = -6x$ y' is squared y is cubed

a) constant b) linear c) quadratic d) cubic e) quartic

$f(x) = [\sin(3-x)]^2$

10) If $f(x) = \sin^2(3 - x)$ then $f'(0) =$

$f'(x) = 2[\sin(3-x)](\cos(3-x))(-1)$

$f'(x) = -2\sin(3-x)\cos(3-x)$

a) -2 cos 3 b) -2 sin 3 cos 3 c) 6 cos 3 d) 2 sin 3 cos 3 e) 6 sin 3 cos 3

$f'(0) = -2\sin 3 \cos 3$

(5,3) $m = -2$

11) If $f(5) = 3$ and $f'(5) = -2$, find the derivative of $x^2 f(x)$ at $x = 5$.

product rule $x^2 f'(x) + f(x)\, 2x$

$(5)^2 f'(5) + f(5)\, 2(5)$

$25(-2) + 3(10)$

a) 0 b) -18 c) -12 d) -20 e) -80

$-50 + 30 = -20$

12) $y = -\frac{1}{\sqrt{x^2+1}}$, then $\frac{dy}{dx} =$

$(\sqrt{x^2+1})^2$ $\frac{\sqrt{x^2+1}}{x^2+1}$

$= \boxed{\frac{x}{(x^2+1)^{3/2}}}$

a) $\frac{x}{(x^2+1)^{1/2}}$ b) $\frac{x}{(x^2+1)^{3/2}}$ c) $\frac{-x}{(x^2+1)^{1/2}}$

d) $\frac{-x}{(x^2+1)^{3/2}}$ e) $\frac{x}{x^2+1}$

chain rule $y = -1(x^2+1)^{-1/2}$

$y' = \frac{1}{2}(x^2+1)^{-3/2}(2x)$

$= \boxed{\frac{x}{(x^2+1)^{3/2}}}$

---

13) If $y = \frac{3}{\sin x + \cos x}$, find $\frac{dy}{dx} =$

quotient rule: $\frac{(\sin x+\cos x)(0) - (3)(\cos x - \sin x)}{(\sin x+\cos x)^2} =$

$\frac{-3(\cos x - \sin x)}{(\sin x+\cos x)^2}$

a) $3\sin x - 3\cos x$ b) $\frac{3}{(\sin x + \cos x)^2}$ c) $\frac{-3}{(\sin x + \cos x)^2}$

d) $\frac{3(\cos x - \sin x)}{(\sin x + \cos x)^2}$ e) $\frac{3(\sin x - \cos x)}{1 + 2\sin x \cos x}$

chain rule: $y = 3(\sin x + \cos x)^{-1}$

$y' = -3(\sin x + \cos x)^{-2}(\cos x - \sin x)$

$= \frac{-3(\cos x - \sin x)}{(\sin x + \cos x)^2}$

$\sin^2 x \; 2\sin x\cos x + \cos^2 x$

$1 + 2\sin x \cos x$

---

14) If $x^2 + xy - y = 2$, find $\frac{dy}{dx}$

Implicit differentiation $2x + x\frac{dy}{dx} + y - 1\frac{dy}{dx} = 0$

$(x-1)\frac{dy}{dx} = -2x - y$

a) $\frac{2x+y}{1-x}$ b) $\frac{2x}{1-x}$ c) $\frac{2x-2}{1-x}$ d) $\frac{2-2x}{x}$ e) DNE

$\frac{dy}{dx} = \frac{-2x-y}{x-1}$

$= \frac{-(2x+y)}{-(-x+1)}$

---

15) If $\sin y = \cos x$, then find $\frac{dy}{dx}$ at the point $\left(\frac{\pi}{2}, \pi\right)$

a) -1 b) 0 c) 1 d) $\frac{\pi}{2}$ e) None of these

$\cos y \frac{dy}{dx} = -\sin x$

$\frac{dy}{dx} = \frac{-\sin x}{\cos y} = \frac{-\sin(\pi/2)}{\cos(\pi)} = \frac{-1}{-1} = 1$

$-\frac{\sqrt{3}}{3}$ $\frac{\sqrt{3}}{3}$

16) If $f(x) = x^3 - x$, then

a) $x = \frac{\sqrt{3}}{3}$ locates a relative maximum of f

b) $x = \frac{\sqrt{3}}{3}$ locates a relative minimum of f

c) $x = \sqrt{3}$ locates a relative maximum of f

d) $x = \sqrt{3}$ locates a relative minimum of f

e) $x = -\sqrt{3}$ locates a relative minimum of f

---

17) $f(x) = x^n$, where n is a positive integer ≥ 2. The graph of $f(x)$ will have an inflection point when n is

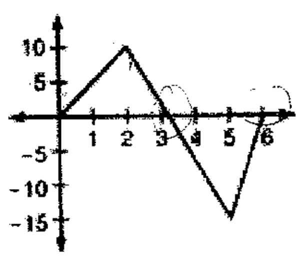
a) even b) odd c) divisible by 3 d) for all values e) for no values

$f(x) = x^2$ $f'(x) = 2x$ $f''(x) = 2$ $f(x) = x^5$ $f'(x) = 5x^4$ $f''(x) = 20x^3$

$f(x) = x^3$ $f'(x) = 3x^2$ $f''(x) = 6x$

---

18) A particle is subjected to gravity according to the following graph of the particle's velocity.



- When velocity = 0, the particle has stopped
- When velocity change from positive to negative (or negative to positive) the particle changes direction

At what time is the particle at its highest point?

a) 0 b) 2 c) 3 d) 5 e) 6

In questions 19-20, a particle moves along a horizontal line according to the formula

$$s = 2t^4 - 4t^3 + 2t^2 - 1$$

particle moves right means velocity is positive

19) The particle is moving right when

$V = 8t^3 - 12t^2 + 4t$

$8t^3 - 12t^2 + 4t > 0$

a) $0 < t < \frac{1}{2}$ b) $t > 0$ c) $t > 1$ d) $0 < t < \frac{1}{2}, t > 1$ e) never

$4t(2t^2 - 3t + 1) > 0$

$(2t - 1)(t - 1)$ $t = 1/2, 1$

$+$ $-$ $+$ $1/2$ 1

---

20) The acceleration, a is increasing when

$a = 24t^2 - 24t$

$a' = 48t - 24$ $t = 1/2$ $-$ $+$ $1/2$

a) $t > 1$ b) $t > 0.5$ c) $t < 0.211$ or $t > 0.789$ d) $0 < t < 0.5$ e) $0 < t < 1$

21) The circumference of a circle is increasing at a rate of $\frac{2\pi}{5}$ inches per minute. When the radius is 5 inches, how fast is the area of the circle increasing in square inches per minute?

a) $\frac{1}{5}$ b) $\frac{\pi}{5}$ c) 2 d) 2π e) 25π

$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$\frac{2\pi}{5} = 2\pi \frac{dr}{dt}$

$1/5 = dr/dt$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(5)\left(\frac{1}{5}\right)$

$\frac{dA}{dt} = 2\pi$

---

22) A conical tank has a height that is always 3 times ~~its radius~~. If water is leaving the tank at the rate of 50 cubic feet per minute, how fast is the water level falling in feet per minute when the water is 3 feet high? Volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

$h = 3r$ $\frac{h}{3} = r$ $\frac{dV}{dt} = -50$ $\frac{dh}{dt} = ?$ $h = 3$

a) 1.000 b) 5.305 c) 15.915 d) 0.589 e) 1.768

$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$

$V = \frac{1}{3}\pi \frac{h^2}{9} h$

$V = \frac{1}{27}\pi h^3$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$

$-50 = \frac{1}{9}\pi (3)^2 \frac{dh}{dt}$

$-50 = \pi \frac{dh}{dt}$

$-50/\pi = dh/dt$

$\frac{dh}{dt} = -15.915$

AP exam may or may not use negative since they told you the rate is falling and didn't just say changing

---

23) A function $f(x)$ is continuous for all x and has a local minimum at (1, 8). Which must be true?

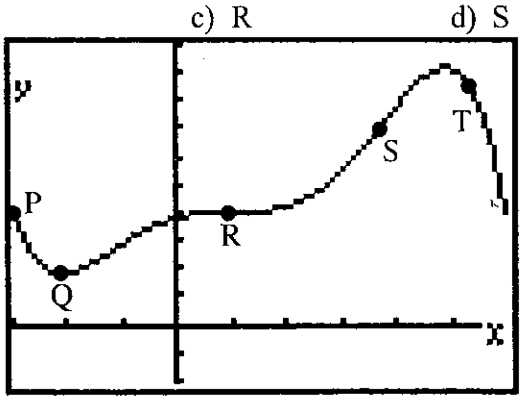
a) $f'(2) = 0$ b) f' exists at $x = 2$ c) the graph is concave down at $x = 1$

d) $f'(x) < 0$ if $x < 1$, $f'(x) > 0$ if $x > 1$ e) $f'(x) > 0$ if $x < 1$, $f'(x) < 0$ if $x > 1$

---

24) At what point on the graph $y = f(x)$ below are both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both less than zero?

a) P b) Q c) R d) S e) T



$\frac{dy}{dx} < 0 \rightarrow$ y dec.

$\frac{d^2y}{dx^2} < 0 \rightarrow$ y concave down

$\frac{6}{-K} = \frac{1}{2}$ $\quad$ $K = 12$

25) If $\lim_{x\to\infty} \frac{6x^2}{200-4x-kx^2} = \frac{1}{2}$, then $k =$

a) 3 b) -3 c. 12 d. -12 e. -3

---

26) For the following piecewise function, determine the answer that best describes it:

$$f(x) = \begin{cases} x - \cos(x), x \ge 0 \\ x^2 + x - 1, x < 0 \end{cases}$$

$f(0) \begin{cases} 0 - \cos(0) = -1 \\ 0^2 + 0 - 1 = -1 \end{cases}$ continuous $\quad$ $f'(0) \begin{cases} 1 + \sin x = 1 \\ 2x + 1 = 1 \end{cases}$ diff

$f(0) = -1$ $\quad$ $f'(0) = 1$

a) both continuous and differentiable b) neither continuous nor differentiable c) continuous only

d) differentiable only e) has a cusp point at $x = 0$

---

27) Let $f(x)$ be a polynomial function such that $f(3) = 3$, $f'(3) = 0$, and $f''(3) = -3$. What is the point (3,3) on the graph $y = f(x)$?

X Y m=0 cc down

a) relative maximum b) relative minimum c) intercept

d) inflection point e) none of these

---

28) Given that $f(x) = 4 + \frac{3}{x}$ find all values of c in the interval (1,3) that satisfy the mean value theorem.

$4 + 3x^{-1}$

continuous/diff on interval

a) 2 b) $\sqrt{2}$ c) $\sqrt{3}$ d) $\pm\sqrt{3}$ e) MVT doesn't apply

$\frac{f(b)-f(a)}{b-a} = f'(x)$

$\frac{f(3)-f(1)}{3-1} = -3x^{-2}$

$\frac{5-7}{2} = \frac{-3}{x^2}$

$-1 = \frac{-3}{x^2}$ $\quad$ $x^2 = 3$

$x = \sqrt{3}$

---

29) $\int (x^2 - 4\sec x \tan x)\, dx =$ $\quad$ $\frac{x^3}{3} - 4\sec x + C$

a) $2x - 4\tan x + C$ b) $\frac{x^2}{3} - 4\tan x + C$ c) $\frac{x^2}{3} - 4\sec^2 x + C$

d) $\frac{x^2}{3} - 4\sec x + C$ e) none of these

30) An equation of the line tangent to $y = \sin x + 2\cos x$ at $\left(\frac{\pi}{2}, 1\right)$ is

a) $2x - y = \pi - 1$
b) $2x + y = \pi + 1$
c) $2x - 2y = 2 - \pi$
d) $4x + 2y = 2 - \pi$
e) none of these

$y' = \cos x - 2\sin x$
$y' = \cos\frac{\pi}{2} - 2\sin\frac{\pi}{2}$
$= 0 - 2(1) = -2$

$y - 1 = -2\left(x - \frac{\pi}{2}\right)$
$y - 1 = -2x + \pi$
$2x + y = \pi + 1$

---

31) $\int \sqrt{x}\left(\sqrt{x} + 1\right) dx = \int x + \sqrt{x} = \frac{x^2}{2} + \frac{2}{3}x^{3/2} + C$

a) $2\left(x^{3/2} + x\right) + C$
b) $\frac{x^2}{2} + x + C$
c) $\frac{1}{2}\left(\sqrt{x} + 1\right)^2 + C$
d) $\frac{x^2}{2} + \frac{2x^{3/2}}{3} + C$
e) $x + 2\sqrt{x} + C$

---

32) For $f(x) = x^{\frac{2}{3}}\left(x^2 - 4\right)$ on [-2 2] the "c" value that satisfies Rolle's Theorem is $f(c)$ exists

a) 0
b) 2
c) ± 2
d) There is no value for c because $f(0)$ does not exist
e) There is no value for c because $f(x)$ is not differentiable on (-2,2) Sharp point at $x=0$

$f(x) = x^{8/3} - 4x^{2/3}$
$f'(x) = \frac{8}{3}x^{5/3} - \frac{8}{3}x^{-1/3} = 0$
$\frac{8}{3}x^{-1/3}\left(x^2 - 1\right)$ $x^{-1/3} = 0$ $\frac{1}{x^{1/3}} = 0$ will never happen

---

33) If $f(x)$ is a continuous function with $f''(x) = -5x^2(2x-1)^2(3x+1)^3$, find the set of values of x for which $f(x)$ has an inflection point. $x=0$ $x=1/2$ $x=-1/3$

a) $\left\{0, -\frac{1}{3}, \frac{1}{2}\right\}$
b) $\left\{-\frac{1}{3}, \frac{1}{2}\right\}$
c) $\left\{-\frac{1}{3}\right\}$
d) $\left\{\frac{1}{2}\right\}$
e) no inflection points

$+ \quad - \quad - \quad -$
$-1/3 \quad 0 \quad 1/2$

34) The smallest slope of $f(x) = 6x^2 - x^3$ for $0 \le x \le 6$ occurs at $x =$

a) 0 b) 2 c) 3 d) 4 e) 6

$f'(x) = 12x - 3x^2$
$f'(0) = 0$
$f'(2) = 12$
$f'(3) = 9$
$f'(4) = 0$
$f'(6) = -36$

---

35) Find the equation of the line tangent to $y = \tan 2x$ at $x = \frac{\pi}{8}$.

$y = \tan 2(\pi/8)$
$y = \tan \pi/4 = 1$
$(\pi/8, 1)$

a) $y - 1 = \sqrt{2}\left(x - \frac{\pi}{8}\right)$ b) $y - 1 = \frac{1}{2}\left(x - \frac{\pi}{8}\right)$ c) $y - 1 = \frac{1}{4}\left(x - \frac{\pi}{8}\right)$

d) $y - 1 = 2\left(x - \frac{\pi}{8}\right)$ e. $y - 1 = 4\left(x - \frac{\pi}{8}\right)$

$y - 1 = 4(x - \pi/8)$

$y' = \sec^2 2x \; (2)$
$y'(\pi/8) = [\sec 2(\pi/8)]^2 (2) = 4$

$f(x) = 2x^6 - 15x^4$

36) Given $f(x) = x^4(2x^2 - 15)$. On what interval(s) is the graph of f concave upwards?

a) $\left(0, \sqrt{3}\right)$ b) $\left(-\sqrt{3}, 0\right)$ c) $\left(-\sqrt{3}, 0\right) \cup \left(0, \sqrt{3}\right)$

d) $\left(-\sqrt{3}, \sqrt{3}\right)$ e) $\left(-\infty, -\sqrt{3}\right) \cup \left(\sqrt{3}, \infty\right)$

$f'(x) = 12x^5 - 60x^3$
$f''(x) = 60x^4 - 180x^2$
$60x^2(x^2 - 3) \quad x = 0, \pm\sqrt{3}$

$+ \quad - \quad - \quad +$
$-\sqrt{3} \quad 0 \quad \sqrt{3}$

---

37) The graph of the function $y = x^5 - x^2 + \sin x$ has a point of inflection at $x =$

$\boxed{\text{calculator}}$

a) 0.324 b) 0.499 c) 0.506 d) 0.611 e) 0.704

$y' = 5x^4 - 2x + \cos x$
$y'' = 20x^3 - 2 - \sin x = 0$
$x = .499$

---

38) If $f'(x) = 2(3x+5)^4$, then the fifth derivative of $f(x)$ at $x = \frac{-5}{3}$ is

a) 0 b) 144 c) 1,296 d) 3,888 e) 7,776

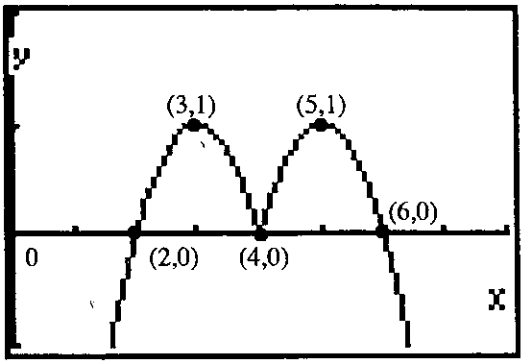
$f^{IV}(x) = 432(3x+5)'(3) = 1296(3x+5)$
$3888x + 6480$

$f''(x) = 8(3x+5)^3(3) = 24(3x+5)^3$
$f'''(x) = 72(3x+5)^2(3) = 216(3x+5)^2$

$f^5(x) = 3888$
$f^5(-5/3) = 3888$

f(x) min where f'(x) changes from neg to pos

Problems 39 and 40 refer to the graph below.



39) The figure shows the graph of $f'(x)$, the derivative of the function f. The domain of f is $0 \le x \le 8$. For what value(s) of x does the function have a relative minimum?

a) 2 b) 4 c) 6 d) 2 and 6 e) 3 and 5

---

40) In what interval(s) is the graph of f concave up? f(x) ccu where f'' pos, f' inc

a) (3, 5) b) (2, 6) c) $[0,3)\cup(4,5)$ d) always e) never

---

41) If $f'(x) = \sin x$ and $f(\pi) = 3$, then $f(x) =$

a) $\cos x + 4$ b) $\cos x + 3$ c) $-\cos x + 2$ d) $-\cos x - 2$ e) $-\cos x + 4$

$\int \sin x = -\cos x + C$ $-\cos \pi + 2$

$-\cos \pi + C = 3$

$1 + C = 3$ $C = 2$

---

42) Let f be a function such that $\lim_{h\to 0}\frac{f(5+h)-f(5)}{h} = 4$. Which of the following must be true?

to have a derivative, f(x) must be continuous

I. $f(5) = 4$

II. $f'(5) = 4$

III. f is continuous at $x = 5$

a) I only b) II only c) III only d) I and II only e) II and III only

43) If $y = 3x - 7, x > 0$, what is the minimum product of x^2y?

$y' = 3$

a) -5.646 b) 0 c) 1.556 d) 2.813 e) 4.841

$x^2\frac{dy}{dx} + y(2x)$

$x^2(3) + (3x-7)(2x)$

$3x^2 + 6x^2 - 14x = 0$

$9x^2 - 14x = 0$

$x(9x-14) = 0$

$x = 0, \ 14/9$

Chain rule

44) Let *f*, *g* and their derivatives be defined by the table below. What is the derivative of $f(g(x))$ at $x = 2$?

x	1	2	3	4
$f(x)$	3	0	1	2
$g(x)$	2	3	4	1
$f'(x)$	1	2	3	1
$g'(x)$	2	2	1	1

$f'g(x)\,g'(x)$

$f'g(2)\,g'(2)$

$f'(3)\,g'(2)$

$3 \cdot 2 = 6$

a) 6 b) 2 c) 8 d) 4 e) 0

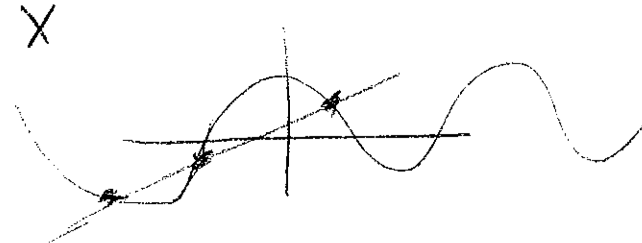
---

45) For how many values of x will the tangent lines to $y = 4\sin x$ and $y = \frac{x^2}{2}$ be parallel? $\boxed{\text{Calculator}}$

a) 0 b) 1 c) 3 d) 4 e) infinite

$y' = 4\cos x$ $y' = x$

$4\cos x = x$



Be sure all your scantron marks are clear and that erasures are complete. Be sure your name is on your scantron. Lightly circle the problem numbers of problems not answered.